The Reputation Politics of Filibustering: Appendix

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1 Strategies and Beliefs

Each party's type is denoted $\theta_i \in \Theta = \{m, e\}$. Information sets for parties are singletons. The four initial information sets for the majority are represented by the four possible permutations of $(\theta_M, \theta_O) \in \Theta^2$. There are four possible information sets for the opposition represented by the four possible permutations of $(\theta_O, \theta_M, \text{bill})$ where the last term "bill" is included for completeness and denotes that the majority submitted a bill. There are four additional information sets for the majority after a bill has been submitted and the opposition filibusters represented by the four permutations of $(\theta_M, \theta_O, \text{bill}, \text{filibuster})$.

A pure strategy for the majority is made up of a bill proposal strategy

$$b : \Theta^2 \to \{\text{bill}, \text{no bill}\}$$

and a strategy in response to a filibuster

$$w : \Theta^2 \times \{\text{bill}\} \times \{\text{filibuster}\} \to \{\text{fight}, \text{table}\}$$

A pure strategy for the opposition is a filibuster strategy

$$f : \Theta^2 \times \{\text{bill}\} \to \{\text{filibuster}, \text{allow vote}\}$$

Own type is the first argument in a party's strategy. To economize on notation I do not define legislator beliefs. Legislators observe (θ_M, θ_O) at the beginning of the game and retain these beliefs at each information set (on and off the equilibrium path).

Constituencies do not take actions in the game. They simply form beliefs after all legislative action ends. For the constituencies, there are five possible information sets. Four of these information sets are singletons. The fifth contains two histories. To distinguish information sets for constituencies from information sets for parties, I italicize information sets for constituencies. Constituencies can observe no bill proposed by the majority, *no bill*, or an unobstructed vote allowed on a bill, *vote*. If the legislative action stage proceeds along either of these paths, with probability $1-\zeta$ the constituencies only know that there is no change to the status quo, *sq*. The remaining two information sets are one in which the majority tables a bill after the opposition initiates a filibuster, *table*, and one in which the majority fights a filibuster, resulting in a visible *filibuster*. Let Φ denote the set of all information sets. Now let

$$\mu_i : \Phi \to [0, 1]$$

denote the constituency i's belief that party i is extreme.

2 Equilibrium refinement

2.1 Equilibrium strategies

I restrict attention to pure strategy equilibria. I further restrict attention to equilibria in which strategies are mappings only from a party's own type into actions on the equilibrium path. More specifically, I impose the following requirement on equilibrium strategies: at information sets reached with positive probability, behavior strategies are mappings from own type only. Restriction of equilibria implies that in equilibrium, constituencies only learn information about each party from its own actions. Neither party can signal the other party's type through its actions alone. Formally, this restriction on strategies states

1)

$$b^*(e, m) = b^*(e, e)$$

$$b^*(m, m) = b^*(m, e)$$

2) If $b^*(\theta_M, \theta_O) =$ "bill" for all $(\theta_M, \theta_O) \in \Theta^2$,

$$f^*(e, e, \text{bill}) = f^*(e, m, \text{bill})$$

$$f^*(m, e, \text{bill}) = f(m, m, \text{bill})$$

3) If $b^*(\theta_M, \theta_O) =$ "bill" and $f^*(\theta_O, \theta_M) =$ "filibuster" for all $(\theta_M, \theta_O) \in \Theta^2$,

$$w^*(e, e, \text{bill}, \text{filibuster}) = w^*(e, m, \text{bill}, \text{filibuster})$$

$$w^*(m, e, \text{bill}, \text{filibuster}) = w^*(m, m, \text{bill}, \text{filibuster})$$

2.2 Off-path beliefs

I impose the following restrictions on constituency off-path beliefs.

If the opposition is not permitted to take any action, the opposition constituency retains it prior belief. This is consistent with my strategy refinement that rules out direct signaling of the opponent's type on the equilibrium path.

- $\mu_O(no\, bill) = 1/2$

If only the extreme opposition filibusters a bill that it expects will be fought in equilibrium, the opposition constituency believes that the opposition is extreme if it observes that a filibuster is initiated. Consistent with my strategy refinement that rules out direct signaling of the opponent's type on the equilibrium path, this prevents the majority from directly signaling the opposition's type by tabling a bill when it is supposed to filibuster.

- If *allow vote* and *filibuster* are reached with positive probability in equilibrium and $\mu_O(allow\ vote) = 0$ and $\mu_O(filibuster) = 1$, then $\mu_O(table) = 1$.

If the extreme majority introduces a bill and fights a filibuster, the majority constituency believes the majority is moderate if it observes that a bill is tabled. This assumes that if the extreme majority is willing to bear the cost of a filibuster fight while the moderate majority is not, the majority constituency forms the same beliefs about a majority that refuses to fight regardless of how it refuses to fight.

- If *filibuster* is reached with positive probability in equilibrium and $\mu_M(filibuster) = 1$, then $\mu_M(table) = 0$.

If one party plays the same strategy at every information set on the equilibrium path, its constituency retains its prior belief at all off-path information sets. This both prevents one party from directly signaling its opponent's type and rules out equilibria that are held together merely by punitive off-path beliefs.

- If $\mu_O(\phi) = 1/2$ for all information sets ϕ reached with positive probability in equilibrium, then $\mu_O(\phi') = 1/2$ for all information sets ϕ' off the equilibrium path.
- If $\mu_M(\phi) = 1/2$ for all information sets ϕ reached with positive probability in equilibrium, then $\mu_M(\phi') = 1/2$ for all information sets ϕ' off the equilibrium path.

If only the moderate majority submits no bill in equilibrium, the majority constituency believes the majority is extreme if it observes a filibuster fight or an allowed vote. This prevents the opposition from directly manipulating the majority constituency's belief by allowing a vote on a bill proposed by the extreme majority. It also rules out an implausible case in which the majority constituency believes that the majority is a moderate type if it observes a filibuster when it knows that the moderate majority is unwilling to fight a filibuster.

- If $b^*(e,m) = b^*(e,e) =$ bill and $b^*(m,m) = b^*(m,e) =$ no bill, then $\mu_M(\phi) = 1$ for $\phi \in \{allow\ vote,\ filibuster\}$.

2.3 Robustness to perturbations

I ignore equilibria that only exist along knife edges in the parameter space. Formally, let $\rho = (\alpha, \beta, p, q, \zeta)$ be a vector of parameters. I restrict attention to equilibria such that if an equilibrium exists for some ρ, it also exists for some $\rho \pm \epsilon$. This rules out equilibria in which at least one type of one party is necessarily indifferent between playing its equilibrium strategy and deviating.

I also ignore equilibria in which at least one type of the majority is indifferent between playing its equilibrium strategy and submitting no bill. This rules out intuitively unappealing equilibria in which the majority takes extra steps to receive the same payoff. For example, it rules out equilibria in which both types of the majority submit a bill, both types of opposition filibuster, and both types of the majority table.

3 Filibuster Equilibrium Definitions

Definition 1 (PB Equilibrium) *In a PB equilibrium*

$$b^*(e,m) = b^*(e,e) = b^*(m,e) = b^*(m,m) = \textit{bill}$$

$$f^*(e,e,\textit{bill}) = f^*(e,m,\textit{bill}) = f^*(m,e,\textit{bill}) = f^*(m,m,\textit{bill}) = \textit{filibuster}$$

$$w^*(e,e,\textit{bill},\textit{filibuster}) = w^*(e,m,\textit{bill},\textit{filibuster}) = \textit{fight}$$

$$w^*(m,e,\textit{bill},\textit{filibuster}) = w^*(m,m,\textit{bill},\textit{filibuster}) = \textit{fight}$$

$$\mu_M(\phi) = 1/2 \quad \textit{for all} \quad \phi$$

$$\mu_O(\phi) = 1/2 \quad \textit{for all} \quad \phi$$

Definition 2 (OS Equilibrium) *In an OS equilibrium,*

$$b^*(e,m) = b^*(e,e) = b^*(m,e) = b^*(m,m) = \textit{bill}$$

$$f^*(e,e,\textit{bill}) = f^*(e,m,\textit{bill}) = \textit{filibuster}$$

$$f^*(m,e,bill) = f^*(m,m,bill) = allow\ vote$$

$$w^*(e,e,bill,filibuster) = w^*(m,e,bill,filibuster) = fight$$

$$w^*(e,m,bill,filibuster) = w^*(m,m,bill,filibuster) = fight$$

$$\mu_M(\phi) = 1/2 \quad for\ all \quad \phi$$

$$\mu_O(no\,bill) = \mu_O(sq) = 1/2$$

$$\mu_O(vote) = 0$$

$$\mu_O(filibuster) = \mu_O(table) = 1$$

Definition 3 (MS Equilibrium) *In a MS equilibrium*

$$b^*(e,m) = b^*(e,e) = bill$$

$$b^*(m,m) = b^*(m,e) = no\ bill$$

$$f^*(e,e,bill) = f^*(e,m,bill) = f^*(m,e,bill) = f^*(m,m,bill) = filibuster$$

$$w^*(e,m,bill,filibuster) = w^*(e,e,bill,filibuster) = fight$$

$$w^*(m,m,bill,filibuster) = w^*(m,e,bill,filibuster) = table$$

$$\mu_M(no\,bill) = \mu_M(table) = \mu_M(sq) = 0$$

$$\mu_M(vote) = \mu_M(filibuster) = \mu_M(table) = 1$$

$$\mu_O(\phi) = 1/2 \quad for\ all \quad \phi$$

Definition 4 (MS-OS Equilibrium) *In a MS-OS equilibrium,*

$$b^*(e,m) = b^*(e,e) = \textit{bill}$$

$$b^*(m,e) = b^*(m,m) = \textit{no bill}$$

$$f^*(e,e,\textit{bill}) = f^*(e,m,\textit{bill}) = f^*(m,m,\textit{bill}) = \textit{filibuster}$$

$$f^*(m,e,\textit{bill}) = \textit{allow vote}$$

$$w^*(e,e,\textit{bill},\textit{filibuster}) = w^*(e,m,\textit{bill},\textit{filibuster}) = \textit{fight}$$

$$w^*(m,e,\textit{bill},\textit{filibuster}) = w^*(m,m,\textit{bill},\textit{filibuster}) = \textit{table}$$

$$\mu_M(\textit{no bill}) = \mu_M(\textit{table}) = \mu_M(\textit{sq}) = 0$$

$$\mu_M(filibuster) = \mu_M(\textit{vote}) = 1$$

$$\mu_O(\textit{no bill}) = \mu_O(\textit{sq}) = 1/2$$

$$\mu_O(\textit{vote}) = 0$$

$$\mu_O(filibuster) = \mu_O(\textit{table}) = 1$$

4 Proofs of Claims in Main Text

Equilibrium existence proofs for Lemmas 1-4 are straightforward. I identify equilibrium and deviation payoffs at each information set and provide a condition for equilibrium payoffs to weakly exceed deviation payoffs. If all conditions are satisfied, the equilibrium exists. If any condition is not satisfied, the equilibrium does not exist. I examine all conditions and find those that are potentially binding. In my proofs below I present only constraints that are not trivially satisfied. Complete proofs are available upon request.

Lemma 1 (PB Equilibrium) *A PB equilibrium exists if and only if* $pq \geq c_O$ *and* $(1-p)q \geq c_M$.

Proof of Lemma 1:

The following inequalities prevent deviations at specified information sets.

Information sets (m,m, bill, filibuster)/(m,e, bill, filibuster)/(m,m)/(m,e): $(1-p)q \geq c_M$

Information sets (m,e, bill)/(m, m, bill): $pq \geq c_O$

These are the two conditions stated in the Lemma. □

Lemma 2 (OS Equilibrium) *An OS equilibrium exists if and only if either*

1) Constituencies are extreme and

$$(1-p)q \geq c_M + \beta$$

$$pq + \alpha \leq c_O$$

2) Constituencies are moderate and

$$(1-p)q + (1-\zeta)\beta \geq c_M$$

$$pq - \alpha \in [0, c_O]$$

Proof of Lemma 2:

I first identify equilibrium conditions at each information set for moderate constituencies:

Information sets (m,m, bill, filibuster)/(m,e, bill, filibuster): $(1-p)q + (1-\zeta)\beta \geq c_M$

Information sets (e, e, bill)/(e, m, bill): $pq - \alpha \geq 0$

Information sets (m, e, bill)/(m, m, bill): $pq - \alpha \leq c_O$

Information set (m,e): $(1-p)q + \beta \geq c_M$

Information sets (e,m)/(m,m): $q - \beta \geq 0$

The condition at information set (m,e) implies the condition at information sets (m,e, bill, filibuster) and (m,m, bill, filibuster). The assumption that $q \geq \alpha + 2\beta$ implies that the condition required at information sets (m,m) and (e,m) is satisfied. This leaves the three conditions stated in the Lemma for a moderate constituencies.

Equilibrium conditions at each information set for extreme constituencies are:

Information sets (m,m, bill, filibuster)/(m,e, bill, filibuster): $(1-p)q - (1-\zeta)\beta \geq c_M$

Information sets (e,m, bill, filibuster)/(e,e, bill, filibuster): $(1-p)q - (1-\zeta)\beta \geq 0$

Information sets (m,m, bill)/(m,e, bill): $pq + \alpha \leq c_O$

Information set (e,e): $(1-p)q - \beta \geq 0$

Information set (m,e): $(1-p)q - \beta \geq c_M$

(m,e) implies (e,e) and (e,m, bill, filibuster)/(e,e, bill, filibuster) and (m,m, bill, filibuster)/(m,e, bill, filibuster). These conditions are those stated in the Lemma for a extreme constituencies. □

Lemma 3 (MS Equilibrium) *A MS equilibrium exists if and only if* $pq \geq c_O$ *and either*

1) Constituencies are extreme and $c_M \geq (1-p)q + \alpha$

2) Constituencies are moderate and $(1-p)q - \alpha \in [0, c_M]$

Proof of Lemma 3

I first identify equilibrium conditions at each information set for extreme constituencies.

Information set (m, m, bill, filibuster)/(m, e, bill, filibuster): $(1-p)q + \alpha \leq c_M$

Information set (m, e, bill): $pq \geq c_O$

These are the two conditions stated in the Lemma for a extreme constituencies.

I now identify equilibrium conditions at each information set for moderate constituencies.

Information sets (e, m, bill, filibuster)/(e,e, bill, filibuster)/(e,e)/(e,m): $(1-p)q \geq \alpha$

Information sets (m, e, bill, filibuster)/(m, m, bill, filibuster): $\alpha + c_M \geq (1-p)q$

Information set (m, e, bill): $pq \geq c_M$

These are the three conditions stated in the Lemma for moderate constituencies. □

Lemma 4 (MS-OS Equilibrium) *A MS-OS equilibrium exists if and only if constituencies are extreme, $c_O \geq \alpha + pq$, and $c_M \geq (1-p)q + \alpha - \beta(1-\zeta)$.*

Proof of Lemma 4:

I first show that a MS-OS equilibrium does not exist if constituencies are moderate. Equilibrium requires that the majority does not submit a bill at information set (m,e) i.e. when it is moderate and the opposition is extreme. Its payoff from playing its equilibrium strategy is $r(k_M(0), 1/2)$. If it instead submits a bill and tables the bill after the majority filibusters, it earns a payoff of $\zeta r(k_M(0), 0) + (1-\zeta) r(k_M(0), 1/2)$. Equilibrium therefore requires $-\zeta\beta \geq 0$. For $\zeta, \beta > 0$, this condition cannot be satisfied.

Now I identify equilibrium conditions for extreme constituencies.

Information sets (e,e, bill, filibuster)/(e,m, bill, filibuster): $(1-p)q + \alpha - \beta(1-\zeta) \geq 0$

Information sets (m, e, bill, filibuster)/(m, m, bill, filibuster): $(1-p)q+\alpha-\beta(1-\zeta) \leq c_M$

Information set (m,e, bill): $c_O \geq pq + \alpha$

Information set (e,e): $(1-p)q + \alpha - \beta \geq 0$

If the condition for (e,e) is satisfied, this implies that the condition for (e,e, bill, filibuster) and (e,m, bill, filibuster) is satisfied. This leaves the three conditions stated in the Lemma. □

Lemma 5 *The unique filibuster equilibrium is*

- *PB if c_M and c_O are sufficiently low.*
- *MS-OS if c_M and c_O are sufficiently high.*
- *MS if c_M is sufficiently high and c_O is sufficiently low.*
- *OS if c_M is sufficiently low and c_O is sufficiently high.*

Proof of Proposition 5: Lemmas 1-4 imply that 1) if $c_O < pq - \alpha$ and $c_M < (1-p)q - \alpha$, the unique filibuster equilibrium is PB; 2) if $c_O \geq \alpha + pq$ and $c_M > \max\{(1-p)q + \alpha -$

$\beta(1-\zeta),(1-p)q+\beta(1-\zeta)\}$ the unique filibuster equilibrium is MS-OS; 3) if $c_O < pq-\alpha$ and $c_M \geq (1-p)q+\alpha$ the unique filibuster equilibrium is MS; 4) if $c_O \geq pq+\alpha$ and $c_M < (1-p)q-\beta$ the unique filibuster equilibrium is OS. □

Proposition 1 *The Senate is supermajoritarian if c_O is sufficiently low.*

Proof of Proposition 1: USMVs occur with positive probability only in MS-OS and OS equilibria. From Lemma 4, if $c_O < pq+\alpha$, the MS-OS equilibrium does not exist. From Lemma 2, the OS equilibrium does not exist if $c_O < pq-\alpha$. □

Proposition 2 *The equilibrium that maximizes the probability of a filibuster exists only if c_O and c_M are sufficiently low.*

Proof of Proposition 2: The probability of a filibuster is maximized in a PB equilibrium. From Lemma 1, a PB equilibrium exists if and only if $c_O \leq pq$ and $c_M \leq (1-p)q$. □

Proposition 3 *The equilibrium that maximizes the probability of a filibuster does not exist if the opposition is sufficiently strong or sufficiently weak.*

Proof of Proposition 3:

The probability of a filibuster is maximized in a PB equilibrium. From Lemma 1, a PB equilibrium does not exist if $p \leq \frac{c_O}{q}$ or $p \geq 1-\frac{c_M}{q}$. □

Proposition 4 *An equilibrium in which filibusters and USMVs occur with positive probability can exist for any p.*

Proof of Proposition 4: Filibusters and USMVs occur with positive probability in a MS-OS equilibrium. If follows from Lemma 4 that the MS-OS equilibrium can exist for any $p \in [0,1]$. □

Proposition 5 *If the opposition is sufficiently weak, a filibuster equilibrium exists only if constituencies are extreme.*

Proof of Proposition 5: From Lemma 1, a PB equilibrium does not exist if $p < \frac{c_O}{q}$. From Lemma 2, if constituencies are moderate an OS equilibrium does not exist if $p < \frac{\alpha}{c_O}$. If $p < \frac{\alpha}{c_O}$ and constituencies are extreme, a OS equilibrium exists if $pq + \alpha \leq c_O$ and $(1-p)q \geq c_M + \beta$. From Lemma 3, a MS equilibrium does not exist if $p < \frac{c_O}{q}$. From Lemma 4, a MS-OS equilibrium does not exist if constituencies are moderate. If constituencies are extreme, Lemma 4 implies that a MS-OS equilibrium can exist for any p. □

Proposition 6 *If the cost of filibustering to both parties is sufficiently high, a filibuster equilibrium exists only if constituencies are extreme.*

Proof of Proposition 6:The proof of Lemma 5 shows that if $c_O \geq \alpha + pq$ and $c_M > \max\{(1-p)q + \alpha - \beta(1-\zeta), (1-p)q + \beta(1-\zeta)\}$, PB, OS, and MS equilibria do not exist. If these conditions are satisfied, a MS-OS equilibrium exists if and only if constituencies are extreme. □

5 Non-existence of other filibuster equilibria

In this section I show that the four filibuster equilibria defined above are the only pure strategy filibuster equilibria that can exist under my equilibrium refinement. A filibuster equilibrium is defined as an equilibrium in which filibusters are initiated with positive ex ante probability. This immediately rules out any equilibrium in which either 1) neither type of majority party submits a bill and 2) both types of opposition allow a vote. It will be useful to say that a party "separates" at an information set if the moderate type takes an action that ends the legislative stage and the extreme type does not. That is, the majority "separates" if the extreme type submits a bill while the moderate type does not or the extreme majority fights a filibuster while the moderate majority tables. Similarly, the opposition separates if the extreme type filibusters and the moderate type allows a vote. I say that a party "backward separates" at an information set if the extreme type takes the action that ends the legislative stage.

It can be shown that there is no equilibrium in which a party backward separates, although proving this in general is somewhat tedious. A complete proof is available upon request. The intuition for the argument is illustrated by considering an equilibrium in which the moderate majority is supposed to fight a filibuster and the extreme majority is supposed to table a bill. At the information set where they are supposed to backward separate, the moderate majority earns an equilibrium payoff of u_m^* and the extreme majority an equilibrium payoff of u_e^*. The extreme majority's deviation payoff is u_m^* while the moderate majority's deviation payoff is $u_e^* + c_M$. Equilibrium therefore requires $u_m^* \geq u_e^*$ to prevent deviation by the moderate type and $u_e^* \geq u_m^* + c_M$ to prevent deviation by the extreme type. That is, equilibrium requires $0 \geq c_M$.

Having ruled out equilibria with backward separation, there are ten classes of equilibria in which filibusters are initiated with positive ex ante probability. Classes of equilibria are distinguished by equilibrium strategies on the equilibrium path of play. Classes may contain more than one strategy profile if multiple strategies at off-path information sets are consistent with an equilibrium path of play. The first four of these classes contain the PB, OS, MS, and MS-OS equilibria respectively. It is straightforward to check that these four classes satisfy the equilibrium refinement as applied to strategies on the equilibrium path and off-path beliefs. Lemmas 1-4 imply that these equilibria are robust to perturbations of parameters and that neither type of majority is necessarily indifferent between submitting a bill and not submitting a bill. My restriction to candidate equilibrium profiles in which filibusters are initiated with positive probability rules out equilibria in which the majority pools on submitting no bill and the opposition pools on allowing a vote. To economize on language, in the descriptions of classes of equilibria below I say that a party simply "pools" if the majority pools on bill introduction or the opposition pools on filibustering. The ten classes are:

1) Majority pool, opposition pool, majority pool on fight.

2) Majority pool, opposition separate, majority pool on fight if extreme opposition fili-

busters.

3) Majority separate, opposition pool if extreme majority introduces bill, extreme majority fights.

4) Majority separate, opposition separate if extreme majority introduces a bill, extreme majority fights if extreme opposition filibusters.

5) Majority pool, opposition pool, majority pool on table.

6) Majority pool, opposition pool, majority separate.

7) Majority pool, opposition separate, majority pool on table if extreme opposition filibusters

8) Majority pool, opposition separate, majority separate if extreme opposition filibusters

9) Majority separate, opposition pool if extreme majority introduces bill, extreme majority tables.

10) Majority separate, opposition separate if extreme majority introduces a bill, extreme majority tables if extreme opposition filibusters.

I first show that the PB, OS, MS, and MS-OS equilibria defined above are the unique equilibria in their classes. I then proceed by showing that equilibria in classes 5-10 cannot exist under my refinement.

1) Majority pool, opposition pool, majority pool on fight.

All information sets for which party strategies must be specified are reached with positive probability in equilibrium. Any deviation by any party ends the legislative stage. Therefore the PB equilibrium is the unique equilibrium in this class.

2) Majority pool, opposition separate, majority pool on fight if extreme opposition filibusters.

The only information sets that are not reached with positive probability in this class of equilibria are (e, m, bill, filibuster) and (m,m, bill, filibuster). That is, the class of equilibrium strategies does not pin down the strategy for either type of majority when the moderate opposition filibusters. Because both types of majority fight the extreme majority's filibuster, the existence of an equilibrium in this class implies that that payoff to each type of majority from fighting exceeds its payoff from tabling:

$$(1-p)q + r(k_M(1/2), k_O(1)) - c_M \geq \zeta r(k_M(1/2), k_O(1)) + (1-\zeta) r(k_M(1/2), k_O(1/2))$$

and

$$(1-p)q + r(k_M(1/2), k_O(1)) > \zeta r(k_M(1/2), k_O(1)) + (1-\zeta) r(k_M(1/2), k_O(1/2))$$

The latter condition implies that there is no equilibrium in which the extreme majority tables a bill off path. This leaves only the possibility that the extreme majority fights off path while the moderate majority tables. This requires a condition that is not robust to perturbations of the model's parameters:

$$(1-p)q + r(k_M(1/2), k_O(1)) - c_M = \zeta r(k_M(1/2), k_O(1)) + (1-\zeta) r(k_M(1/2), k_O(1/2))$$

Therefore the OS equilibrium is the only equilibrium in this class that survives my refinement.

3) Majority separate, opposition pool if extreme majority introduces bill, extreme majority fights.

This class of equilibria does not specify strategies when the moderate majority submits a bill. Strategies must be defined off path for the opposition's response to this deviant behavior and the moderate majority's response if the opposition filibusters a bill. The MS equilibrium prescribes that the opposition filibusters and the moderate majority tables. It is straightforward to check that there is no equilibrium in which the opposition allows a vote. In equilibrium, the moderate opposition weakly prefers to engage in a costly filibuster fight with the extreme majority and the extreme opposition strictly prefers to engage the extreme majority in a filibuster fight. If the moderate majority fights a filibuster, the opposition's payoffs from filibustering and allowing a vote are equivalent to its payoffs when confronting a bill proposed by the extreme majority. The extreme opposition therefore must filibuster. The moderate opposition can allow a vote only if it is exactly indifferent between allowing a vote and filibustering. Such an equilibrium strategy can be sustained only on a knife edge and therefore does not survive the refinement if the moderate majority fights. If the moderate majority tables, equilibrium strategies in response to the extreme majority imply that the opposition strictly prefers to filibuster. Therefore in any equilibrium in this class that survives my refinement, both types of the opposition filibuster if the moderate majority submits a bill.

Now consider the moderate majority's response to a filibuster. If the moderate majority fights it earns a payoff of $(1-p)q + r(k_M(1), k_O(1/2)) - c_M$. If it tables its payoff is $\zeta r(k_M(0), k_O(1/2)) + (1-\zeta)r(k_M(0), k_O(1/2)) = r(k_M(0), k_O(1/2))$. Therefore the following inequality must be satisfied for the moderate majority to fight a filibuster off path: $(1-p)q \pm \alpha \geq c_M$ where $\pm\alpha$ depends on whether constituencies are moderate or extreme. Now consider the moderate majority's decision to submit a bill or not. Existence of equilibrium implies that payoff from no bill weakly exceeds payoff from submitting a bill and fighting an inevitable filibuster: $(1-p)q \pm \alpha \leq c_M$. Thus in order for the majority to play its equilibrium strategy of submitting no bill and fight a filibuster, the moderate majority must be exactly indifferent between submitting a bill and not submitting a bill. Such an equilibrium is therefore not robust to parameter perturbations. It follows that the MS equilibrium is the only equilibrium in this class that survives my refinement.

4) Majority separate, opposition separate if extreme majority introduces a bill, extreme majority fights if extreme opposition filibusters.

To show that the MS-OS equilibrium is the only equilibrium in its class under my refinement, the following off-path strategies must be ruled out:

$w^*(e, m, \text{bill}, \text{filibuster}) = \text{table}$

$w^*(m, e, \text{bill}, \text{filibuster}) = w^*(m, m, \text{bill}, \text{filibuster}) = \text{fight}$

$f^*(e, m, \text{bill}) = f^*(m, m, \text{bill}) = \text{allow vote}$

It is straightforward to rule out any equilibrium in which the extreme majority tables a bill if the moderate opposition filibusters i.e. any equilibrium with $w^*(e, m, \text{bill}, \text{filibuster}) =$ table. The majority's payoffs from fighting and tabling are independent of the opposition's type after the opposition filibusters. Equilibrium requires that the payoff from fighting weakly exceeds the payoff from tabling because equilibrium requires that the extreme majority fights the extreme opposition's filibuster. Thus in order for the majority to table a bill filibustered by the moderate opposition, the extreme majority must be exactly indifferent between tabling and filibustering a bill. An equilibrium with this strategy therefore is not robust to a perturbation of the model's parameters.

Now consider an equilibrium in which the moderate majority fights a filibuster instead of tabling a bill after the opposition initiates a filibuster. The majority's payoff is independent of the opposition's type after the opposition has initiated a filibuster. Therefore its strategy at information set $(m, e, \text{bill}, \text{filibuster})$ must be the same as its strategy at information set $(m, m, \text{bill}, \text{filibuster})$ unless tabling and fighting yield exactly the same payoff. This implies that a strategy that proscribes a different action at each information set is not robust to parameter perturbations. Consider then an equilibrium in which the majority fights a filibuster. Equilibrium requires that the moderate opposition does not filibuster if the majority fights the filibuster: on the equilibrium path of play, it allows a vote when the extreme majority submits a bill because the extreme majority fights any filibuster as established above. It follows that unless the moderate opposition is exactly indifferent between allowing a vote and enduring a filibuster fight, the moderate opposition must allow a vote if the moderate majority submits a bill. Given this response, if the moderate majority submits a bill to a moderate opposition, it earns the same payoff as the extreme majority does from submitting a bill to the moderate opposition. Unless the extreme majority is indifferent between submitting no bill and submitting a bill to the moderate majority, the moderate majority strictly prefers to submit a bill to the moderate majority. But this violates an equilibrium strategy on the equilibrium path proscribed in this class of equilibria. Therefore the moderate majority must table a bill that is filibustered in any equilibrium in this class that is robust to parameter perturbations.

Finally, consider an equilibrium in which the opposition allows a vote on a bill submitted by the moderate majority. Equilibrium requires that the extreme majority prefers to submit a bill that the moderate opposition allows a vote on to submitting no bill. If the opposition allows a vote on a bill proposed by the moderate majority, the moderate majority earns the same payoff as the extreme majority when it submits a bill that the moderate opposition allows a vote on. An equilibrium in which the opposition allows a vote on a bill proposed by the moderate majority therefore requires that i) the moderate majority is indifferent between submitting a bill and proposing no bill and ii) the extreme majority is indifferent between submitting a bill to the moderate opposition and submitting no bill. Such an equilibrium is therefore not robust to parameter perturbations.

5) Majority pool, opposition pool, majority pool on table.

Under my refinement, constituencies retain their prior beliefs at every information set in any equilibrium in this class. The moderate majority earns an equilibrium payoff of $r(1/2, k_O(1/2))$. If it submits no bill, it also earns a payoff of $r(1/2, k_O(1/2))$. That is, neither action results in a change to the status quo and both admit the same beliefs by constituencies as equilibrium. The majority is exactly indifferent between submitting a bill and not submitting a bill. Therefore no equilibrium in this class survives my refinement.

6) Majority pool, opposition pool, majority separate.

If the moderate majority does not submit a bill, the opposition constituency retains its prior belief about the opposition just as it does in equilibrium. The moderate majority earns no policy payoff in equilibrium and also no policy payoff from submitting no bill. If the majority constituency observes that the majority does not submit a bill, it believes that the majority is a moderate type. In equilibrium, regardless of whether the majority constituency observes *table* or *status quo*, it believes the majority is a moderate type by Bayes' rule. Therefore the moderate majority earns the exact same payoff from submitting no bill as it does from playing its equilibrium strategy. The equilibrium therefore does not survive my refinement.

7) Majority pool, opposition separate, majority pool on table if extreme opposition filibusters

Equilibrium requires that the extreme majority is better off tabling the bill than fighting the filibuster. Fighting the filibuster yields a gain in policy payoff of $(1-p)q$ relative to equilibrium. The extreme majority does not pay an opportunity cost to fight. Reputation payoffs must therefore prevent deviation by the extreme majority. Specifying deviation payoffs requires that MC and OC beliefs are defined at the off-path information set *filibuster*. Intuitively, the OC should believe the opposition is an extreme type because only the extreme opposition initiates a filibuster. My refinement requires this. Therefore the only effect of a deviation by the extreme type on its payoff is the payoff from own reputation. In equilibrium the MC believes the majority is an extreme type with probability 1/2. Equilibrium therefore requires that $r(k_M(1/2), k_O(1)) \geq r(k_M(\mu_M(1/2)), k_O(1)) + (1-p)q$ or $\alpha[k_M(1/2) - k_M(\mu_M(1/2))] \geq (1-p)q$. Assume this holds with strict equality i.e. the MC forms sufficiently punishing beliefs for this deviation. In this case, the extreme majority is strictly better off tabling the bill than fighting the filibuster. But now consider the moderate opposition. If the moderate opposition filibusters and the majority tables, it earns the same payoff as the extreme opposition does in equilibrium. If the extreme opposition is strictly better off filibustering (as required by equilibrium) then the moderate majority strictly prefers to deviate if the majority tables. Equilibrium therefore requires that the majority responds to this deviation by fighting. But because the extreme type is strictly better off tabling, this deterrence strategy is not sequentially rational. Therefore if the extreme majority is strictly better off tabling and the extreme majority is strictly better off filibustering, an equilibrium of this class does not exist. Equilibrium requires (at least) that the extreme majority is indifferent between fighting and tabling or the extreme majority is indifferent between filibustering and allowing a vote. Any equilibrium in this class is therefore not robust to small perturbations of p, α, and q.

8) Majority pool, opposition separate, majority separate if extreme opposition filibusters

Equilibrium requires that the moderate majority weakly prefers to table than fight the filibuster. Assume the moderate majority strictly prefers to table. If the moderate opposition filibusters and the majority tables, it earn the same payoff as the extreme opposition. Equilibrium requires that this payoff is weakly greater than the moderate opposition's payoff from allowing a vote. If the extreme opposition strictly prefers to filibuster, then the moderate majority must filibuster to prevent deviation by the moderate opposition. But because the moderate majority strictly prefers to table, this deterrence strategy is not sequentially rational. Equilibrium therefore requires (at least) that either the moderate majority is exactly indifferent between fighting and tabling or the extreme opposition is exactly indifferent between allowing a vote and filibustering. Any such equilibrium is therefore not robust to small perturbations of the model's parameters.

9) Majority separate, opposition pool if extreme majority introduces bill, extreme majority tables.

In any equilibrium with such a strategy profile, if constituencies are moderate, the extreme majority is strictly better off imitating the moderate majority and introducing no bill. In equilibrium, it harms its own reputation, does not affect the opposition's reputation, and receives no policy payoff. If it submits no bill, the OC retains its prior belief, no policy payoff is earned but the extreme majority enhances its reputation. Any such equilibrium therefore requires that the MC is extreme. If constituencies are extreme, however, the extreme majority raises its policy payoff by fighting and does not affect the OC's belief. The MC believes the majority is an extreme type if it observes a filibuster. Therefore the extreme majority weakly prefers to table as required by equilibrium if and only if $(1-p)q = 0$. Any equilibrium with such a strategy profile is therefore not robust to small perturbations of the model's parameters.

10) Majority separate, opposition separate if extreme majority introduces a bill, extreme

majority tables if extreme opposition filibusters.

Under the restriction I place on off-path beliefs, if the extreme majority fights instead of tabling, the MC believes it is an extreme type and the OC believes the opposition is an extreme type. If constituenceis are extreme, the extreme majority improves its own reputation in equilibrium and that of its opponent. It is strictly better off fighting and winning a policy victory with some probability as well unless $q(1-p)=0$. Any such equilibrium therefore does not survive parameter perturbations if constituenceis are extreme. If constituencies are moderate, when the opposition is extreme the extreme majority's equilibrium payoff is $\zeta r(0,0)+(1-\zeta)r(1-\mu_M(sq),1-\mu_O(sq))$. If it instead does not introduce a bill, its deviation payoff is $\zeta r(1,1/2)+(1-\zeta)r(1-\mu_M(sq),1-\mu_O(sq))$. If it introduces a bill but fights the filibuster, its deviation payoff is $(1-p)q+r(0,0)$. Submission of a bill therefore requires $\beta\geq\alpha$ while tabling requires $\alpha[1-\mu_M(sq)]\geq\beta[1-\mu_O(sq)]$. By Bayes' rule, $\mu_M(sq)=\frac{1/4}{1/4+1/2}=1/3$ and $\mu_O(sq)=\frac{1/4}{1/4+1/4}=1/2$. Combining the two conditions yields the necessary condition for equilibrium $3/2\leq 1$. Therefore any equilibrium with such a profile does not exist.