Virtue Signaling: A Theory of Message Legislation

Appendix

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Contents

1 Overview

In Section 2 of this Appendix we provide formal proofs of all claims made in the main text. In Section E, we present the extension to the baseline model discussed in the main text, characterize its equilibria, and use these equilibria to derive the welfare results we present in the main text. In Section M, we present a generalized version of the baseline model. In this generalized model, we provide a micro-foundation for the assumptions we make regarding the bargaining process between the legislator and veto player and the non-strategic behavior of the voter and legislators in the second period.

2 Proofs of All Claims in Main Text

Proposition 1 *A messaging equilibrium exists if and only if* (3) *is satisfied:*

$$\frac{\zeta\phi}{k} \geq \lceil\frac{\zeta b}{k}\rceil - \frac{\zeta b}{k} \tag{3}$$

Proof of Proposition 1: In the main text we show that (3) is a necessary condition for equilibrium. Here we show that (3) is also sufficient. Assume that a messaging equilibrium does not exist. This implies that either the slacker or zealot (or both) can profitably deviate from messaging equilibrium strategies. Given voter beliefs, the slacker cannot be made better off relative to equilibrium if he chooses $n < \lceil\frac{\zeta b}{k}\rceil$. This deviation has no effect on his probability of reelection and costs him nk more than his equilibrium strategy. By construction, if the slacker legislates $n \geq \lceil\frac{\zeta b}{k}\rceil$ times, he is weakly worse off compared to equilibrium. Therefore if a messaging equilibrium does not exist, there must exist a profitable deviation for the zealot. Given voter beliefs off path, deviation to $n > \lceil\frac{\zeta b}{k}\rceil$ raises the cost of legislating to the zealot but does not affect his probability of reelection relative to equilibrium. Voter off-path beliefs also imply that all $n < \lceil\frac{\zeta b}{k}\rceil$ yield the same probability of reelection which is less than his probability of reelection in equilibrium. This implies that his payoff-maximizing deviation is $n = 0$. Non-existence of a messaging equilibrium therefore implies that the zealot is strictly better off imitating the slacker than playing his equilibrium strategy. In the main text we show that (3) implies the zealot is weakly better off playing his equilibrium strategy than legislating zero times. Thus no messaging equilibrium implies (3) fails. Thus if (3) is satisfied, a messaging equilibrium exists. □

Corollary 1 *A messaging equilibrium exists if* $\frac{\zeta\phi}{k} \geq 1$.

Proof of Corollary 1: For any $y \in \mathbb{R}_+$, $\lceil y \rceil - y \in [0, 1)$. Thus $\lceil \frac{\zeta b}{k} \rceil - \frac{\zeta b}{k} < 1$. Therefore $\frac{\zeta\phi}{k} \geq 1$ implies that (3) holds with strict equality. By Proposition 1, this implies that a messaging equilibrium exists. □

Lemma 1 *A messaging equilibrium is the unique separating equilibrium.*

Proof of Lemma 1: We show in the main text that there is no separating equilibrium in which either $n^*(s) > 0$ or $n^*(z) < \lceil \frac{\zeta b}{k} \rceil$. We complete the proof here by formally showing that a separating equilibrium with $n^*(z) > \lceil \frac{\zeta b}{k} \rceil$ does not survive the intuitive criterion. The existence of such an equilibrium implies that the zealot is weakly better off legislating $n^*(z) > \lceil \frac{\zeta b}{k} \rceil$ times and being reelected with probability ζ than not legislating at all and being reelected with probability zero. Were this not the case, he could profitably deviate from equilibrium and imitate the slacker. Because $\lceil \frac{\zeta b}{k} \rceil < n^*(z)$ and $k > 0$, the zealot is strictly better off relative to equilibrium if he legislates $\lceil \frac{\zeta b}{k} \rceil$ times and is reelected with probability ζ. The voter's off-path beliefs must therefore satisfy $\mu(\lceil \frac{\zeta b}{k} \rceil) < \pi^C$ to prevent this deviation. However, $\lceil \frac{\zeta b}{k} \rceil$ is constructed such that the zealot is weakly worse off legislating zero times and losing the election with probability one than legislating $\lceil \frac{\zeta b}{k} \rceil$ and being reelected with probability ζ. Thus under the intuitive criterion, if the voter observes $n = \lceil \frac{\zeta b}{k} \rceil$, she must believe that the incumbent is a zealot. Therefore a separating equilibrium with $n^*(z) > 0$ does not survive the intuitive criterion. □

Proposition 2 (No Legislation Equilibrium) *A no legislation equilibrium exists if and only if either (i) the voter is trusting or (ii) the voter is skeptical and* (4) *is satisfied.*

$$\frac{\zeta\phi}{k} \leq \lceil\frac{\zeta b}{k}\rceil - \frac{\zeta b}{k} \tag{4}$$

Proof of Proposition 2:

It is straightforward to check that (i) is a sufficient condition for equilibrium. If the voter is trusting, both types of incumbents are reelected with probability one in equilibrium and legislate zero times. Their expected payoff is globally maximized. They therefore cannot possibly gain by deviating from equilibrium.

To show sufficiency of (ii), assume (ii) is satisfied. Given off-path voter beliefs, any deviation by either type results in electoral defeat. They are therefore both strictly better off legislating zero times than legislating a positive number of times.

Thus if either (i) or (ii) is true, a no legislation equilibrium exists: (i) and (ii) are sufficient for equilibrium.

We now show necessity: if both (i) and (ii) are false, then a no legislation equilibrium does not exist. If (i) and (ii) are false, this implies that the voter is skeptical and (4) fails. To show that the no legislation equilibrium does not survive the intuitive criterion, assume the voter reelects $n = \lceil\frac{\zeta b}{k}\rceil$ if she is skeptical and (4) fails. The slacker's expected deviation payoff in this counterfactual case is $\zeta b - \lceil\frac{\zeta b}{k}\rceil k$. He is reelected if and only if the voter observes n because she is skeptical and pays $\lceil\frac{\zeta b}{k}\rceil k$ regardless of whether or not the voter observes n. This is strictly greater than her equilibrium payoff of zero if and only if $\frac{\zeta b}{k} > \lceil\frac{\zeta b}{k}\rceil$. By the properties of the ceiling function, this inequality cannot be satisfied. The slacker therefore can never strictly benefit from this deviation. Now consider the zealot's payoff from this deviation. The zealot's expected deviation payoff is $\zeta(b+\phi) - \lceil\frac{\zeta b}{k}\rceil k$. His equilibrium payoff of zero is weakly greater than this deviation payoff if and only if (4) is satisfied. Because

(4) fails, his deviation payoff strictly exceeds his equilibrium payoff. Under the intuitive criterion, the voter must believe that the incumbent is a zealot with probability one if she observes $n = \lceil \frac{\zeta b}{k} \rceil$: $\mu(\lceil \frac{\zeta b}{k} \rceil) = 1$. But the no legislation equilibrium requires $\mu(\lceil \frac{\zeta b}{k} \rceil) < \pi^C$. Therefore if both (i) and (ii) are false, a no legislation equilibrium does not survive the intuitive criterion. □

Lemma 2 *If the voter is skeptical, a no legislation equilibrium is the unique pooling equilibrium.*

Proof of Lemma 2: Consider a pooling equilibrium with $n^*(z) = n^*(s) > 0$. If the voter is skeptical, incumbents lose the election with probability one in any pooling equilibrium. Under the most punishing off-path beliefs at $n = 0$, incumbents lose the election with probability one in equilibrium. Because legislation is costly, both types can profitably deviate to $n = 0$. Their reelection probability remains the same but they save $n^*(z)k$ in wasted legislative effort. Thus no pooling equilibrium with $n^*(z) = n^*(s) > 0$ can exist if the voter is skeptical. □

Lemma 3 *If the voter is skeptical and* (4) *fails, the unique equilibrium is messaging. If the voter is skeptical and* (3) *fails, the unique equilibrium is no legislation.*

Proof of Lemma 3: If (3) is not satisfied, then (4) is satisfied. If (4) is not satisfied, then (3) is satisfied. Proposition 1 establishes that a messaging equilibrium exists if and only if (3) is satisfied. Lemma 1 establishes that the unique separating equilibrium is a messaging equilibrium. Proposition 2 establishes that if the voter is skeptical, a no legislation equilibrium exists if and only if (4) is satisfied. Lemma 2 establishes that if the voter is skeptical, the unique pooling equilibrium is no legislation. □

Proposition 3 *If* $\frac{\zeta \phi}{k} \geq 1$ *and the voter is skeptical, the unique equilibrium is messaging.*

Proof of Proposition 3: From Corollary 1, $\frac{\zeta \phi}{k} \geq 1$ implies that (3) is strictly satisfied which implies that (4) fails. Lemma 3 thus implies that if $\frac{\zeta \phi}{k} \geq 1$, the unique equilibrium is messaging. □

Proposition 4 *From the perspective of an outside observer, if $\pi^C \sim F$, then the probability that a messaging equilibrium must exist and be unique is increasing in*

- *the probability that the voter observes messaging, ζ*
- *the probability that the policy window opens, ρ*
- *the extremity of the status quo, λ*

and decreasing in

- *the voter's prior belief that the incumbent is a zealot, π^I*
- *the cost of non-viable legislation, k*
- *the cost of viable legislation, c*

Proof of Proposition 4: In the main text we show that if $\pi^C \sim F$, the probability that a messaging equilibrium is guaranteed to exist and be unique is

$$F(1 - (\frac{k}{\zeta\rho} + c)\frac{1}{\lambda^2}) - F(\pi^I)$$

A cdf is weakly increasing in its argument. Therefore all else equal, this probability is weakly decreasing in π^I. Note that $1 - (\frac{k}{\zeta\rho} + c)\frac{1}{\lambda^2}$ is decreasing in k and c and increasing in ζ, ρ, and λ^2. Therefore the probability is weakly decreasing in k and c and weakly increasing in ζ, ρ, and λ^2. $\square$

Proposition 5 (Extent of Messaging) *In a messaging equilibrium, the number of times that the zealot legislates is increasing in office benefit, b, and the probability that the voter observes legislative activity, ζ, and decreasing in the cost of conducting non-viable legislation, k.*

Proof of Proposition 5: From Proposition 1, the zealot legislates $n^*(z) = \lceil \frac{\zeta b}{k} \rceil$ in a messaging equilibrium. The ceiling function is weakly increasing in its argument. □

Proposition 6 (Voter Welfare) *The voter's welfare is strictly greater in a messaging equilibrium than a no legislation equilibrium.*

Proof of Proposition 6: In both equilibria, the voter earns the same first-period payoff, $-\lambda^2$. We therefore focus on the voter's welfare in the second period. In a messaging equilibrium, the voter's ex ante expected second-period utility is

$$-\lambda^2[\pi^I(1-\rho) + (1-\pi^I)(1-\rho)\pi^C + (1-\pi^I)(1-\pi^C)]$$

In a no legislation equilibrium with a trusting voter, the voter's ex ante expected utility is

$$-\lambda^2(1-\pi^I\rho)$$

Rearranging terms, welfare is greater in a messaging equilibrium if and only if

$$\pi^I + (1-\pi^I)[(1-\rho)\pi^C + (1-\pi^C)] \leq 1$$

Because $(1-\rho)\pi^C + (1-\pi^C) < 1$, the inequality holds strictly. With a skeptical voter, in a no legislation equilibrium ex ante utility is

$$-\lambda^2(1-\pi^C\rho)$$

An analogous argument establishes that welfare is greater in a messaging equilibrium than a no legislation equilibrium when voters are skeptical. □

Lemma 4 *The voter's welfare is greater in a messaging equilibrium than a no legislation* *equilibrium if and only if*

$$\pi^C(1-\pi^I)\beta(0) \geq \pi^I[\beta(q^*(0)) - \beta(q^*(\lceil \frac{b}{k_n} \rceil))] \tag{7}$$

Proof of Lemma 4: See Lemma E4 below. □

Proposition 7 *The value of messaging to the voter is increasing in*

- *the probability that the challenger is a zealot,* π^C
- *the cost of messaging,* k_n

and decreasing in

- *the ex ante probability that the incumbent is a zealot,* π^I
- *the legislator's office benefit,* b
- *the degree to which messaging and quality investment are substitutes,* k_{nq}

Proof of Proposition 7: See Proposition E1 below. □

E Extension: Quality of Legislation

In this section we present our extension of the baseline model. In the main text we assume, as in the baseline model, that legislators in the second period and voters are non-strategic. Here we allow all players to be strategic. In doing so we show that the assumptions we make about non-strategic legislator and voter behavior in the main text are rationalized by equilibrium strategies in the complete model.

E.1 Setup

In the baseline model we assume that the status quo is repealed and replaced with probability one whenever a legislator chooses to do so if the policy window opens. In this extension we introduce randomness into the policy outcome following an attempt at repeal. In the first period, the legislator selects a level investment in future legislation, $q \in Q = \mathbb{R}_+$. If the legislator attempts to alter the status quo when the policy window opens, this is successful with probability $\beta(q)$. With probability $1 - \beta(q)$, the repeal is unsuccessful and the status quo remains in place. We assume that $\beta(\cdot)$ is strictly increasing and concave with $\beta(0) > 0$ and that investment is specific to a legislator. That is, a zealot who is defeated in period one cannot pass the investment made in the first period on to a replacement. Newly elected challengers successfully change the status quo in the event that the policy window opens with probability $\beta(0)$. If the policy window opens in the second period, the legislator chooses whether to attempt to change the status quo. If he is unsuccessful after this single attempt, the policy window closes.

Investment in a future plan to alter the status quo is costly. Doomed legislative effort moreover is a substitute for investment and raises the marginal cost of investing in future success. For consistency with the baseline model and clarity we assume that the legislator's cost function is

$$c(n, q) = nk_n + qk_q + nqk_{nq}$$

with $k_n, k_q, k_{nq} > 0$ although our results generalize to any cost function that is strictly increasing and weakly convex in both inputs with a positive cross-partial derivative. Let, $n_t \in N = \mathbb{N}$ represent the number of times that a legislator attempts to repeal and replace the status quo in legislative period $t \in \{1, 2\}$. To economize on notation we assume that an attempt to replace the status quo in the second period costs k_n, although our results generalize to a second-period cost of viable legislation that is distinct from the cost of non-viable legislation. Voters observe the extent of legislation in the first period but do not observe the legislator's investment in future legislation. This investment is private non-verifiable information. As in the baseline model the status quo is λ, office benefit is b, the incumbent is a zealot with probability π^I, the challenger a zealot with probability π^C, and policy window opens with probability ρ. For simplicity we assume that the voter always observes legislative effort ($\zeta = 1$). The legislator and voter's ideal points and policy loss functions remain the same as in the baseline model.

The sequence of play is illustrated in Figure 1. First, Nature draws the incumbent legislator's type. The legislator then selects legislative effort and quality investment, (n_1, q). The voter observes n_1 and reelects the incumbent or elects the challenger. After the election, Nature draws the challenger's type and determines whether the policy window opens or not. In the second period, the legislator decides whether or not to legislate. If the legislator declines to legislate, λ remains policy. If the legislator chooses to legislate, Nature determines whether the final policy outcome is 0 or the status quo, λ. With probability $\beta(q)$, the policy outcome is 0. Otherwise, the status quo, λ, remains in place. Payoffs are realized and the game ends.

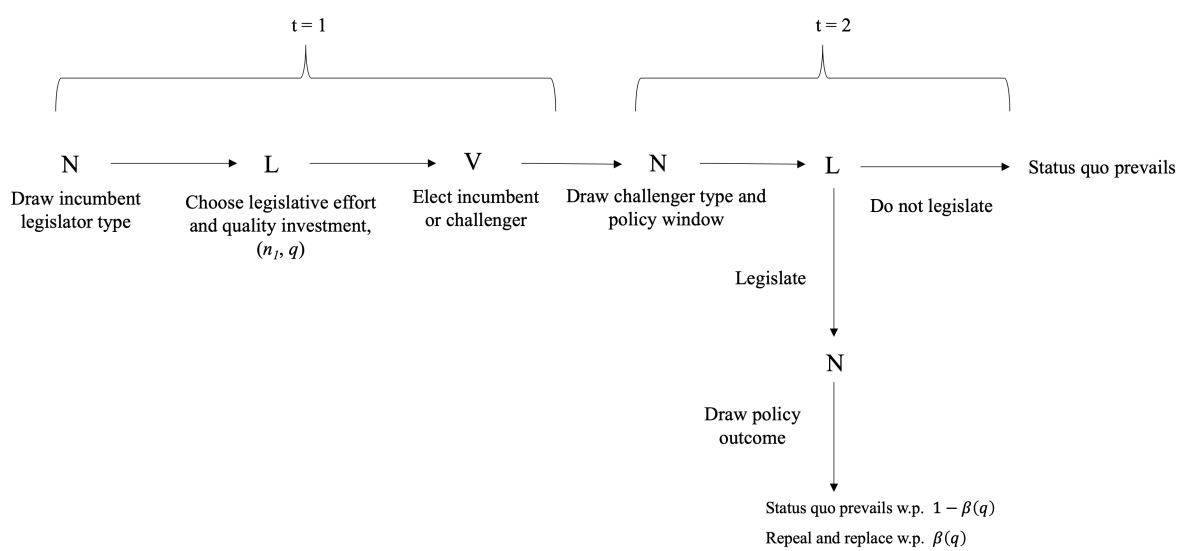

Figure 1: Sequence of play with quality investment

E.2 Extension Strategies and Beliefs

Let $p \in P = \{0, 1\}$ indicate whether or not the policy window is open at the start of the second legislative period. A pure strategy for an incumbent bureaucrat is a mapping

$$m_I : \Gamma \times P \to \mathbb{N}^2 \times Q$$

that returns a triple (n_1, n_2, q). A pure strategy for a challenger is defined similarly as

$$m_C : \Gamma \times P \to \mathbb{N}$$

and returns a second period level of effort, n_2. A pure strategy for the voter is a mapping from the set of all information sets distinguished by the level of effort n_1 she observes,

$$w : \Psi \to \{I, C\}$$

Her beliefs at information set $\psi = n_1$ are represented by a probability distribution over $\Gamma \times Q$, $f(n_1)$. For any $f(n_1)$, let $\zeta(n_1) \equiv Pr(\gamma = z, q = q^*(n_1)|n_1)$ and $\eta(n_1) \equiv Pr(\gamma = s, q = 0|n_1)$ where $q^*(n_1)$ is defined below (and in the main text) as the zealot's optimal level of investment given n_1.

E.3 Equilibrium

We consider weak sequential equilibria that satisfy the intuitive criterion analogous to the messaging and pooling equilibria in the baseline model.

Second-period equilibrium strategies are straightforward. Slackers never legislate. Zealots do not legislate if the policy window remains closed. Both cases result in no change to the status quo. If the policy window opens, zealots legislate if there is a sufficiently high probability of changing the status quo relative to the cost of attempting repeal: $k_n \leq \lambda^2 \beta(q)$. We assume that $k_n \leq \lambda^2 \beta(0)$ so that zealots always legislate if the policy window opens.

Given these strategies, the voter's optimal choice of candidate depends on her beliefs about both the incumbent legislator's type *and* the amount of effort he has expended on developing legislation. In the baseline model, if the voter believes the incumbent is less likely to be a zealot than the challenger, it is always optimal to choose the challenger. Now incumbent zealots are at least as effective at changing policy than new zealots. Accordingly, voters may prefer an incumbent to a challenger even if she believes that the challenger is more likely to be a zealot.

In a messaging equilibrium, the zealot chooses the least amount of visible legislative effort necessary to distinguish himself from the slacker. Because the voter does not observe q, the zealot can only credibly commit to investing $q^*(n_1)$ in the quality of future legislation where

$$q^*(n_1) \equiv \arg\max_q \quad \lambda^2 \rho \beta(q) - qk_q - qn_1 k_{nq}$$

is level of investment that maximizes the zealot's utility when he is retained and legislates for n_1 rounds. It is straightforward to check that $q^*(n_1)$ is decreasing in n_1. The more effort the zealot is required to spend messaging, the less effort he devotes towards developing an effective plan to implement if the policy window opens. In any equilibrium, the slacker invests nothing in future legislation. As in the baseline model, the minimum amount of equilibrium nonviable legislation that the zealot must enact to separate from the slacker is $n^* \equiv \lceil \frac{b}{k} \rceil$. Also as in the baseline model, a messaging equilibrium exists whenever zealots are willing to exert more visible legislative effort to ensure retention than slackers. Formally, this requires an incentive compatibility condition for a messaging equilibrium analogous to (3):

$$b - \rho k_n - q^*(n^*)(k_q + k_{nq} n^*) + \lambda^2 [\beta(q^*(n^*)) - \pi^C \beta(0)] \geq 0 \tag{E1}$$

Lemma E1 (Extension Messaging Equilibrium) *The following is an equilibrium if and only if* (E1) *is satisfied. Legislator's strategies are* $m_I^*(z, p) = (n^*, p, q^*(n^*))$, $m_I^*(s, p) =$

$(0,0,0)$, $m^*_C(z,p) = p$, $m^*_C(s,p) = 0$. The voter's strategy is $w^*(n_1) = 1$ if $n_1 \geq n^*$, $w^*(n_1) = 0$ if $n_1 < n^*$. The voter's beliefs are such that $\zeta(n_1) = 1$ if $n_1 \geq n^*$ and $\zeta(n_1) = 0$ if $n_1 < n^*$.

Proof of Lemma E1: We begin by showing that (E1) is a necessary condition for the equilibrium to exist. Given the strategies specified in the definition, the zealot's equilibrium expected payoff is

$$b - \rho k_n - \lambda^2(1 - \rho\beta(q^*(n^*))) - q^*(n^*)(k_q + k_{nq}n^*)$$

If the zealot instead exerts no legislative effort and is replaced by a challenger, it is optimal for him to also exert no effort investing in future legislation, as all investment goes to waste if he is not reelected. Unless the policy window opens, his replacement is a zealot, and policy implementation is successful, he suffers policy loss. His payoff from this action is therefore

$$-\lambda^2(1 - \rho\pi^C\beta(0))$$

His equilibrium payoff exceeds this if and only if (E1) is satisfied.

We now show that (E1) is a sufficient condition for the messaging equilibrium to exist. It is straightforward to check that second-period legislator strategies are optimal. If the policy window opens, it is optimal for the zealot to repeal and replace the status quo for all q if and only if the policy window opens under the assumption that $k_n \leq \lambda^2\beta(0)$. Slackers do not benefit from policy and therefore maximize their second-period payoff by not legislating.

If the voter elects the challenger, her expected payoff is $-\lambda^2(1 - \rho\pi^C\beta(0))$. Given the beliefs specified in equilibrium, if $n_1 \geq n^*$, her payoff from electing the incumbent is $-\lambda^2(1-\rho\beta q^*(n_1)))$. This exceeds her payoff from electing the challenger. Her strategy for all $n_1 \geq n^*$ is therefore optimal. If $n_1 < n^*$, her payoff from electing the incumbent is $-\lambda^2$ which is lower than her payoff from electing the challenger. Her strategy for all $n_1 < n^*$ is therefore optimal.

For $n_1 = 0$ and $n_1 = n^*$, her beliefs satisfy Bayes' rule. Off path, by the definition of n^*

slackers are strictly worse off than in equilibrium if they exert $n_1 > n^*$. Therefore the voter's belief that the legislator is a zealot with probability one survives the intuitive criterion. Her belief that the zealot chooses q to maximize her payoff given that she invests n_1 is consistent with this belief. For $n_1 < n^*$ off path, the slacker is strictly better off than in equilibrium if he is retained. Therefore the voter's belief that the legislator is a slacker satisfies the intuitive criterion. Her belief that the slacker chooses q to maximize her payoff is consistent with this belief.

Because q is unseen and the slacker does not care about policy, his optimal strategy is to never invest in future legislation. Given the voter's strategy, any number of rounds that results in retention, $n_1 \geq n^*$, harms the slacker. Any number of rounds $n_1 < n^*$ results in the election of the challenger which harms the slacker relative to equilibrium.

Finally, for the zealot any $n_1 > n^*$ results in retention but reduces her payoff both by imposing greater cost in period one and reducing the probability of successful repeal in period two. Any $n_1 < n^*$ results in the election of the challenger. If (E1) is satisfied, no $n_1 < n^*$ (in particular $n_1 = 0$) makes her better off. Given that the exerts n^* in equilibrium, $q^*(n^*)$ is her unique optimal level of investment. Therefore (E1) implies that the equilibrium exists. □

There are two varieties of pooling equilibria, one in which incumbents are reelected and one in which challengers are elected. If incumbents are reelected in equilibrium, zealots invest $q^*(0)$ in developing quality future legislation. That is, they direct all of their attention towards the future and maximize the probability of successfully changing the status quo if the policy window opens. This type of pooling equilibrium exists if voters' prior beliefs about the incumbent's type are sufficiently favorable. Voter beliefs necessary to support a pooling equilibrium in which the voter reelects the incumbent are

$$\pi^I \geq \frac{\pi^C \beta(0)}{\beta(q^*(0))} \tag{E2}$$

Lemma E2 (Pooling Equilibrium with Investment) *The following is an equilibrium if* *and only if* (E2) *is satisfied. The legislators' strategies are* $m_I^*(z,p) = (0, p, q^*(0))$, $m_I^*(s,p) = (0,0,0)$, $m_C^*(z,p) = p$, $m_C^*(s,p) = 0$. *The voter's strategy is* $w^*(n_1) = 1$ *for all* n_1. *On the equilibrium path the voter's beliefs are* $\zeta(0) = \pi^I$, $\eta(0) = 1 - \pi^I$. *Off path,* $\zeta(n_1) = 1$.

Proof of Lemma E2: We show in the proof of Lemma E1 that second-period legislator actions are optimal. Incumbents are reelected and thus maximize their payoff globally in equilibrium. Thus all that needs to be shown to prove necessity is that the (E2) is necessary and sufficient for the voter to reelect the incumbent. On the equilibrium path, $\zeta(0) = \pi^I$ by Bayes' rule. The voter's equilibrium strategy dictates that she reelect the incumbent. This yields an expected payoff of $-\lambda^2(1 - \rho\pi^I\beta(q^*(0)))$. If instead she elects the challenger, her expected payoff is $-\lambda^2(1 - \rho\pi^C\beta(0))$. Rearranging and canceling terms reveals that it is optimal for the voter to reelect the incumbent if and only if (E2) holds. □

In the pooling equilibrium in which voters elect the challenger, zealots invest nothing in the quality of future legislation, as this costly investment will go to waste. Two conditions are necessary for this type of pooling equilibrium to exist. First, voters must be skeptical. Second, under the intuitive criterion slackers must strictly prefer to exert at least as much visible legislative effort to ensure retention than zealots. Otherwise the zealot can signal his type by passing more doomed legislation than the voter knows the slacker is willing to pass if reelected. This is the opposite of the condition required for a messaging equilibrium, (E1). Unless the zealot is indifferent between separating from the slacker and losing the election (which occurs only on a knife-edge in the parameter space), if a messaging equilibrium exists then a pooling equilibrium in which challengers are elected does not exist.

Lemma E3 (Pooling Equilibrium without Investment) *The following is an equilibrium if and only if (i)* $\pi^C > \pi^I$ *and (ii)* (E1) *holds with equality or fails. The legislators' strategies are* $m_I^*(z,p) = (0,p,0)$, $m_I^*(s,p) = (0,0,0)$, $m_C^*(z,p) = p$, $m_C^*(s,p) = 0$. *The voter's strategy is* $w^*(n_1) = 0$ *for all* n_1. *The voters beliefs are* $\zeta(0) = \pi^I$, $\eta(0) = 1 - \pi^I$, *and* $\zeta(n_1) = 1 - \eta(n_1) = 0$ *for all* $n_1 > 0$.

Proof of Lemma E3: We first show that (i) is a necessary condition for equilibrium. On the equilibrium path, the voter believes the that the legislator invested nothing with probability one and is a zealot with probability π^I. Her payoff from reelecting the legislator is therefore $-\lambda^2(1 - \pi^I \rho\beta(0))$. Her equilibrium payoff is $-\lambda^2(1 - \pi^C \rho\beta(0))$ given that she is supposed to elect the challenger. Equilibrium is weakly more valuable than the deviation only if (i) is satisfied.

We now show that (ii) is necessary. If (E1) holds strictly, then the zealot's payoff from legislating $\lceil \frac{b}{k_n} \rceil = n^*$ times is strictly greater than his payoff from no legislation and no investment. From the definition of n^*, the slacker weakly prefers to legislate zero times and lose the election to legislating n^* and winning. Therefore under the intuitive criterion, the voter must believe that the legislator is a zealot and reelect him. This undermines the existence of the pooling equilibrium.

We now show that the conditions are sufficient for equilibrium. We have established earlier that second-period actions are sequentially rational. The voter's equilibrium strategy is optimal given her beliefs and second-period legislator actions if (i) is satisfied. To show that the off-path beliefs are consistent with the intuitive criterion, note that if (ii) is satisfied with equality or fails then the zealot is no better off legislating n^* times and being reelected than he is in equilibrium. From the definition of n^*, the slacker is no better off legislating for n^* times either. Therefore the beliefs specified survive the intuitive criterion for $n_1 = n^*$ i.e. that the legislator is a slacker. For $n_1 > n^*$, the zealot is worse off if (ii) holds. Therefore pessimistic beliefs also survive the intuitive criterion. Finally for $n_1 < n^*$ off path, the slacker is strictly better off legislating $n_1 < n^*$ times if he is retained than in equilibrium. This follows from the definition of n^*. Therefore the specified off-path beliefs are consistent with the intuitive criterion. Given the voter's strategy, neither the slacker nor zealot can do better than investing nothing in future quality and legislating zero times. Regardless of the actions they take, they lose the election. □

As discussed in the main text, because our interest in this extension is a welfare comparison between equilibria with messaging and equilibria without, we focus on voters who have sufficiently generous prior beliefs such that both a pooling equilibrium with reelection and a messaging equilibrium exist. To assess voter welfare with and without messaging, we compare these two types of equilibria.

E.4 Extension Voter Welfare

The voter's welfare can be expressed as the ex ante probability of changing the status quo, conditional on the policy window opening. In a pooling equilibrium, the status quo is changed if the policy window opens with probability

$$\pi^I \beta(q^*(0))$$

In this equilibrium, the voter gambles that the incumbent is a zealot and allows him to devote all of his efforts towards future legislation, foregoing a second draw from the pool of candidates if the incumbent is a slacker. In a messaging equilibrium, the status quo changes if the policy window opens with probability

$$\pi^I \beta(q^*(n^*)) + (1 - \pi^I)\pi^C \beta(q^*(0))$$

In the messaging equilibrium, the voter gets to learn the incumbent's type. Relative to the pooling equilibrium, this yields an added benefit of $(1 - \pi^I)\pi^C \beta(q^*(0))$. If the incumbent is a slacker, the voter gets a second attempt at selecting a zealot by voting for the challenger. If the challenger is a zealot, he successfully changes the status quo if given the opportunity with probability $\beta(q^*(0))$. This is a lower probability of changing the status quo than an incumbent zealot in either the messaging or pooling equilibrium but it is better than the zero probability of changing the status quo that arises in the pooling equilibrium when the incumbent is a slacker. This benefit of screening comes at a cost in terms of the quality of legislation in the event that the incumbent is a zealot. In the messaging equilibrium, the incumbent zealot diverts attention away from his plan to successfully change the status quo in order to signal his type to the voter through failed legislation. This cost in terms of the lost probability of changing the status quo is $\beta(q^*(n^*)) - \beta(q^*(0))$.

Lemma E4 *The voter's ex ante expected payoff is greater in a messaging equilibrium than a pooling equilibrium if and only if*

$$\pi^C(1-\pi^I)\beta(0) \geq \pi^I[\beta(q^*(0)) - \beta(q^*(n^*))] \tag{E3}$$

Proof of Lemma E4:

It follows from Lemma E2 that the voter's ex ante expected payoff in a pooling equilibrium with investment is

$$-\lambda^2(1-\pi^I) + \pi^I[-\lambda^2(1-\rho) + \rho(-\lambda^2(1-\beta(q^*)))]$$

From Lemma E1, the voter's expected payoff in a messaging equilibrium is

$$-\lambda^2[1-\rho(\pi^I\beta(q_s^*) + \pi^C\beta(0)(1-\pi^I))]$$

Setting up the appropriate inequality and rearranging terms yields (E3). □

We can use condition (E3) to identify how the voter's welfare in a messaging equilibrium changes with respect to her welfare in a pooling equilibrium. This tells us how the value of messaging to the voter changes as parameters change.

Proposition E1 *The voter's relative welfare in a messaging equilibrium is increasing in*

- *the expected quality of the challenger,* π^C
- *the cost of legislating,* k_n

and decreasing in

- *the ex ante expected quality of the incumbent,* π^I
- *the legislator's office benefit,* b
- *the degree to which messaging and quality investment are substitutes,* k_{nq}

Proof of Proposition E1: Lemma E4 identifies the condition under which the messaging equilibrium is more valuable than a pooling equilibrium. The solution $q^*(n_1)$ satisfies the first order condition

$$\frac{\partial \beta(q)}{\partial q} = \frac{n_1 k_{nq} + k_q}{\lambda^2 \rho}$$

Because $\beta(\cdot)$ is strictly concave, $\frac{\partial \beta(q)}{\partial q}$ is decreasing in q. It follows that a rise in $\frac{n_1 k_{nq}+k_q}{\lambda^2 \rho}$ yields a decrease in $q^*(n_1)$. We need to compare the relative rates of change of $q^*(0)$ and $q^*(n^*)$. Both terms are increasing in λ and ρ. Without further assumptions on the third derivative of $\beta(\cdot)$, it is ambiguous how $q^*(0) - q^*(n^*)$ changes. Similarly, both terms are decreasing in k_q. Without further assumptions, it is unclear which falls more.

A rise in b on the other hand only affects $q^*(n^*)$. This raises n^* which results in a fall in $q^*(n^*)$. Similarly a rise in k_n leads to a fall in n^* and a rise in $q^*(n^*)$. It has no effect on $q^*(0)$. A rise in k_{nq} does not directly affect n^* but only affects $\beta(n^*)$ by scaling n^* in the numerator of the right-hand side of the first order condition.

Finally note that π^C and π^I have no effect on $q^*(n_1)$ directly or n^*. It follows from Lemma 4 that relative welfare in a messaging equilibrium is increasing in π^C and decreasing in π^I. $\square$

M Micro-founded Model

In this section we present a generalized version of the baseline model in the main text. We model the bargaining process between a strategic legislator and strategic veto player with a compact policy space in both legislative periods. The voter is also a strategic player in this model. We show that the simplifying assumptions we make regarding the binary policy space, non-strategic second-period legislator behavior, non-strategic voter behavior, and reduced-form bargaining are rationalized by the general model. It then follows that our results in the baseline model obtain in the generalized model as well.

M.1 Setup

There are two periods, denoted with subscripts $t \in \{1, 2\}$. The actors are a citizen voter, the voter's representative in the legislature, and a president. There is a one-dimensional policy space, $X = [0, \lambda]$. The voter and president are policy oriented. Define x_t^f as the policy in effect at the end of period t. There is a status quo policy in place at the start of period one, x_0^f, assumed for simplicity to be λ. We normalize the citizen's ideal point to zero. The citizen's payoff in period t is

$$u_t^V(x_t^f) = -(x_t^f)^2$$

and total utility is simply the sum of utilities in the two periods. There is no discounting.

There are two types of presidents, *right* and *left*. A right president's ideal policy is λ and a left president's ideal point is normalized to 0. The president's type is common knowledge. The first-period president is a right president. With probability $\rho \in (0, 1)$, the second-period president is a left president. Formally, denote the president's type $\theta \in \Theta = \{l, r\}$. A right president's payoff in period t is

$$u_t^P(x_t^f; r) = -(x_t^f - \lambda)^2$$

A left president's payoff in period t is

$$u_t^P(x_t^f; l) = -(x_t^f)^2$$

The incumbent president does not value policy after leaving office. We think of the status quo as signature policy such as the Affordable Care Act which a president has no interest in replacing with a weaker law even if this hedges against complete repeal after he or she leaves office.

A legislator is either a slacker or a zealot. Formally, the legislator's type is $\gamma \in \Gamma = \{s, z\}$. A slacker values only the non-policy benefit of holding office, $b > 0$. A zealot additionally values policy. We assume that zealots value policy the same way as the voter with an ideal point of zero and quadratic loss. The legislator's type is private information. An incumbent legislator is a zealot with ex ante probability $\pi^I \in (0, 1)$. A challenger is a zealot with ex ante probability $\pi^C \in (0, 1)$. We define the legislator's complete utility function after describing the sequence of play.

The sequence of play is as follows, illustrated in Figure 2. First Nature selects the incumbent legislator's type. The game then enters a series of rounds of legislative action, each round comprised of legislative action and presidential response. Figure 3 describes the legislative action stage. In this stage, the legislator may pass a piece of legislation, an attempt to "repeal and replace" the status quo. The content of legislation in round ι of period t is a spatial location $x_t^\iota \in X$. Legislating costs the legislator k in time and effort. Given the passage of a bill x_t^ι, the president may veto the legislation or accept it. We assume no recourse to a veto (for example, in a standard veto game the veto-override player is more extreme than the president) so a veto kills the legislation and the status quo x_{t-1}^f continues to prevail. The legislative action stage may continue indefinitely: the legislator and president may interact potentially an infinite number of times. Either of two choices ends the legislative action stage. If the legislator declines to legislate or the president accepts the legislation, the legislative action stage ends. An end to legislation leads to the electoral stage of the game. In the electoral stage, the voter may either retain the sitting legislator or elect a challenger. The voter observes the legislative action stage with probability ζ. Play then enters period two. Nature selects the new president and the challenger's type. Second-period play then consists entirely of another legislative action stage identical to that in period one. Play ends when either the second-period legislator declines to legislate or the second-period president accepts a piece of enacted legislation. Payoffs are realized and the game ends.

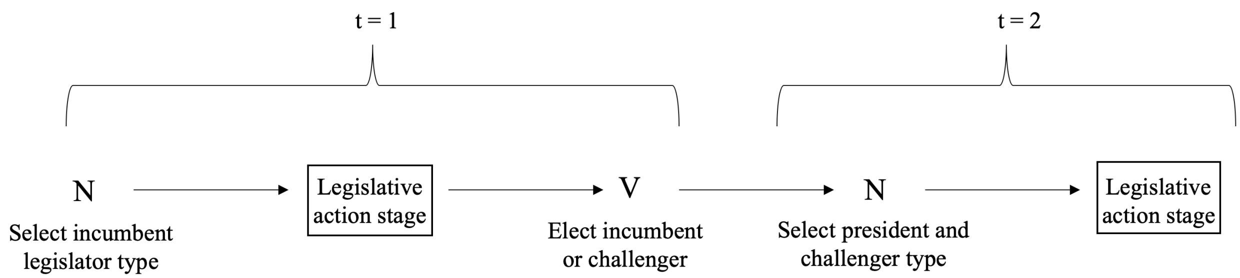

Figure 2: Sequence of play

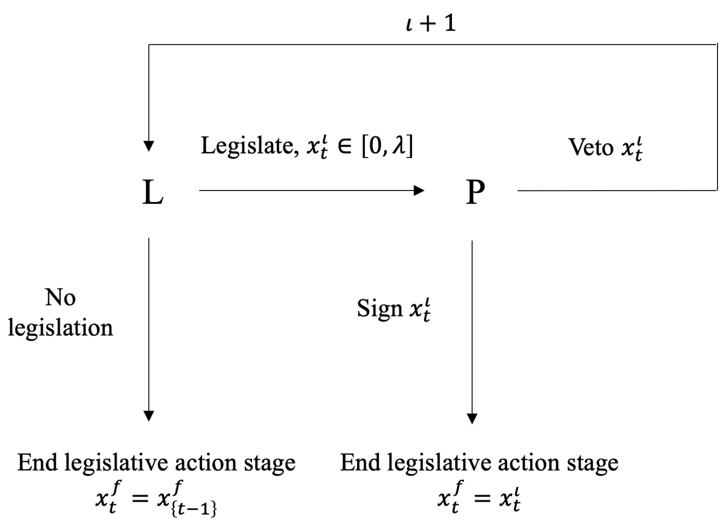

Figure 3: Legislative action stage

Having defined the sequence of the game, it is now straightforward to describe the legislator's utility. Recall that legislative effort is costly and that only zealots value policy. A zealot in office in period t earns a payoff of

$$u_t^L(x_t^f, n_t; z) = b - (x_t^f)^2 - n_t k$$

where n_t is the number of times the legislator passes legislation in the legislative action stage of period t. The slacker's payoff when in office in period t is

$$u_t^L(n_t; s) = b - n_t k$$

We assume that zealots continue to value policy even after removed from office, just as the citizen does. This evaluation distinguishes a zealous legislator from a policy-minded implementor who may avoid some or all of the distress and shame of carrying out a detested policy by quiting the job, a "clean hands" phenomenon. A zealot that loses to a challenger thus receives a second-period payoff of $-(x_2^f)^2$. Unlike zealots, slackers do not care about policy and earn a payoff of zero if out of office.

M.1.1 Strategies and Histories

We denote the legislator's decision to legislate in round ι in period t by $a_t^\iota \in A = \{0, 1\}$ where 0 connotes no legislation and 1 connotes legislating. If the legislator declines to legislate, the result is the null bill, $x_t^\iota = \emptyset$. Denote the president's veto decision as $v_t^\iota \in V = \{0, 1\}$ where 0 denotes acceptance of the legislation and 1 denotes a veto. Denote the voter's electoral decision with $e \in E = \{0, 1\}$ where $e = 0$ connotes the election of a challenger and $e = 1$ the reelection of the incumbent.

We now formally describe a *round* of legislative action. A round of legislative action begins with the legislator's decision to legislate and is indexed by $\iota \geq 1$. The actions taken in a distinct round can fall into one of three categories. First, a legislator can decline to legislate ($a_t^\iota = 0, x_t^\iota = \emptyset$). Note that when $a_t^\iota = 0$, the required bill is $x_t^\iota = \emptyset$. The decision to not legislate ends the legislative action stage. Second, the legislator can legislate ($a_t^\iota = 1$) and pass a bill $x_t^\iota \in [0, 1]$ which the president accepts ($v_t^\iota = 0$). This also ends the legislative action stage. Third, the legislator can legislate ($a_t^\iota = 1$) and pass a bill $x_t^\iota \in [0, 1]$ which the president vetoes ($v_t^\iota = 1$). The conclusion of this third category of round returns the legislative action stage to the legislator and a new round begins. A pure strategy for a legislator at round ι is a prescription of which action to take at each information set given his type. A strategy for the legislator in period t is comprised of a legislative action and bill choice strategy,

$$b_t(h^L, \gamma) : H^L \times \Gamma \to A \times X \cup \emptyset$$

The play in the legislative action stage generates an information set for the voter, I^V. Define the set of all possible information sets for the voter as $\mathcal{I}^V$. Each information set contains two histories, one in which the legislator is a zealot and one in which he is a slacker. With probability $1 - \zeta$, the voter does not observe the legislative action stage. With probability ζ, she observes all the actions taken by the president and legislator in the legislative action stage. Formally, if the voter observes the legislative stage, she observes a sequence of the form $h^V = \{a_1^1, x_1^1, v_1^1, a_1^2, x_1^2, v_1^2, ...\}$. Define $\bar{n} \in \mathbb{N}$ to be the index number of the final round of the legislative phase. If the legislative stage concludes in acceptance of the bill, h^V ends with $a_1^{\bar{n}} = 1$, $x_1^{\bar{n}} \in [0, \lambda]$, $v_1^{\bar{n}} = 0$. If the legislative phase concludes with the legislator declining to legislate, h^V ends with $a_1^{\bar{n}} = 0$, $x_1^{\bar{n}} = \emptyset$. All rounds $\iota < \bar{n}$ are necessarily of the form $a_1^{\iota} = 1$, $x_1^{\iota} \in [0, \lambda]$, $v_1^{\iota} = 1$. That is, the legislator passes a bill and the bill is vetoed. If the voter does not observe the legislative action sequence, she observes $h^V = \emptyset$. Let H^V denote the set of all possible sequences of legislative action (including the null sequence). Every $h^V \in H^V$ corresponds to a unique information set. Every information set corresponds to a unique $h^V \in H^V$. A pure strategy for the voter is a mapping from each information set into her action space, E. Given the bijection between H^V and $\mathcal{I}^V$, this mapping can be expressed as

$$w(h^V) : H^V \to E$$

Denote the voter's belief that the incumbent legislator is a zealot at information $h^V \in H^V$ set as $\mu(h^V)$.

It will be useful to define the *length* of a legislative action stage. Recall that n_t refers to the number of times a legislator in period t passes legislation. Note that $n_t \in \{\bar{n}, \bar{n} - 1\}$. We refer to a sequence h^V with n_t rounds for which $a_1^\iota = 1$ as a legislative sequence *of length* n. Let $\chi(h^V)$ be a function that returns the length of h^V and let $\chi(\emptyset) = \emptyset$.

In the legislative phase of period t, the legislator's information sets are all singletons (the legislator knows his type). A history for the legislator in round ι is a sequence of the form

$$h_t^L = \{a_t^1, x_t^1, v_t^1, a_t^2, x_t^2, v_t^2, ...a_t^{\iota-1}, x_t^{\iota-1}, v_t^{\iota-1}\}$$

By the structure of the game, $a_t^{\hat{\iota}} = 1$, $x_t^{\hat{\iota}} \in [0, 1]$, and $v_t^{\hat{\iota}} = 1$ for all $\hat{\iota} < \iota$. That is, in all previous rounds the legislator must have passed legislation which was vetoed. Let H^L denote the set of all possible histories in a period for the legislator.

To ease notation, we assume that the president knows the legislator's type. The incumbent president earns no payoff after period one. Accepting any bill $x_1^\iota < \lambda$ is therefore strictly dominated by vetoing. For $x_1^\iota = \lambda$, the president is indifferent between accepting and vetoing. In principle, the president can condition his strategy when $x_1^\iota = \lambda$ on the legislator's type which provides information to the voter. To rule out this possibility, we restrict attention to equilibria in which the president vetoes when indifferent. To economize on notation, we do not introduce notation to express the president's belief about the legislator's type.

In the legislative phase of period t, all of the president's information sets, $I^P \in \mathcal{I}^P$ are singletons. At each information set, the president observes a unique sequence, $h_t^P = \{a_t^1, x_t^1, v_t^1, a_t^2, x_t^2, v_t^2, ...a_t^\iota, x_t^\iota\}$. By the structure of the game, $a_t^{\hat{\iota}} = 1$, $x_t^{\hat{\iota}} \in [0, 1]$, and $v_t^{\hat{\iota}} = 1$ for all $\hat{\iota} \leq \iota$. That is, the legislator must have passed legislation which the president vetoed in all previous rounds and the legislator must have passed a bill in the current round. Let H^P denote the set of all possible sequences in a period for the president. A pure strategy for the president in period t is a mapping from his type and the set of information sets into V. Given the bijection between H^P and $\mathcal{I}^P$, this mapping can be expressed as

$$p_t(h^p, \theta) : H^P \times \Theta \to V$$

To focus on the most plausible cases, we make the following assumption.

Assumption 1 $k < \lambda^2(1-\pi^C)$

In our model zealots are policy motivated. Assumption 1 ensures that they are sufficiently policy motivated to work to change the status quo in the event that the policy window opens. If the condition in Assumption 1 is not satisfied, zealots prefer to pass off responsibility for changing policy to a replacement legislator. While an interesting possibility to consider, we do not believe that this type of freeriding is plausible in most cases. If our assumption is incorrect, then zealots are willing to voluntarily lose reelection in the hope that their replacement will do the policy work for them. Under our assumption, zealots are sufficiently policy motivated such that they always prefer to ensure that a zealot (themselves) is in office in the event that the policy window opens rather than shirk and hope that their replacement is a zealot.

M.2 Equilibrium

Our equilibrium concept is a weak sequential equilibrium that satisfies the intuitive criterion. We restrict attention to pure strategies and only consider equilibria in which the president vetoes if indifferent and the voter reelects if indifferent. We begin with period two then consider period one, constructing separating and pooling equilibria.

Second-period equilibrium strategies are straightforward. In the second period, under united government the zealous legislator proposes the citizen activist's preferred bill which the president immediately signs into law unless an acceptable policy was enacted in the first period, in which case there is no legislation. Under divided government, the zealous legislator does not legislate and the status quo from the first period prevails.

Lemma M1 (Second-period Equilibrium Outcomes) *If the status quo is not replaced in the first period, in the second period of any equilibrium*

- *slackers never legislate*

- *if the policy window opens, zealots pass their ideal bill which the president signs immediately*

- *if the policy window remains closed, zealots do not legislate*

Proof of Lemma M1: We first formally state the equilibrium strategies that generate the outcomes described in the Lemma:

If the second-period president is left, second-period equilibrium strategies are

$$b_2^*(h^L; z) = \begin{cases} (1, 0) & \forall h^L \quad \text{if} \quad k \leq (x_1^f)^2 \\ (0, \emptyset) & \forall h^L \quad \text{if} \quad k > (x_1^f)^2 \end{cases}$$

$$b_2^*(h^L; s) = (0, \emptyset) \quad \forall h^L$$

$$p_2^*(h^P; l) = \begin{cases} 0 & \text{if} \quad x_2^\iota < x_1^f \\ 1 & \text{if} \quad x_2^\iota \geq x_1^f \end{cases}$$

If the second-period president is right, second-period equilibrium strategies are

$$b_2^*(h^L; z) = b_2^*(h^L; s) = (0, \emptyset) \quad \forall h^L$$

$$p_2^*(h^P; r) = \begin{cases} 1 & \text{if} \quad x_2^\iota \leq x_1^f \\ 0 & \text{if} \quad x_2^\iota > x_1^f \end{cases}$$

We now prove the Lemma. A left president's ideal point is 0. Therefore if the president is left and the status quo is x_1^f, it is a best response to sign any $x_2^\iota \leq x_1^f$ and veto any $x_2^\iota \geq x_1^f$. Similarly it is a best response for the right president to sign any $x_2^\iota \geq x_1^f$ and veto any $x_2^\iota \leq x_1^f$. It costs the legislator k to pass legislation. At round ι, it is only valuable to pass legislation x_2^ι if the legislation will be accepted and the policy benefits of acceptance exceed k. Because slackers receive no policy utility from any policy, their unique optimal strategy is to never pass legislation. If the president is right, any policy that can be passed makes the zealot worse off. Therefore if the president is right, the zealot's uniquely optimal strategy is to never pass legislation. If the president is left, any policy that is beneficial to the zealot is passed. Therefore if he passes policy, it must be his maximally beneficial policy, $x_2^\iota = 0$. This is sufficiently beneficial to offset cost k if and only if $k \leq (x_1^f)^2$. □

Lemma M1 provides a micro-foundation for our assumption in the baseline model that zealots non-strategically legislate and successfully change the status quo if and only if the policy window opens while slackers do not legislate.

Equilibrium strategies for the voter and first-period president are similarly straightforward. The voter reelects the incumbent legislator if and only if she believes the challenger is less likely to be a zealot than the incumbent. Because the status quo in period one is equal to the incumbent president's ideal point, the incumbent president can never gain by signing legislation. In fact he is strictly worse off signing any $x_1^f < \lambda$. It is therefore an equilibrium strategy for the president to veto all legislation.

Given these strategies, a slacker earns a payoff in period two of b if reelected and earns a payoff of zero if the challenger is elected. If the zealot is reelected, his payoff in the second period is

$$W \equiv b - \rho k - \lambda^2(1 - \rho)$$

He earns an office benefit, b, regardless of whether or not the policy window opens. If the policy window opens, he expends k in legislative effort to move policy from λ to his ideal point, 0. If the policy window remains closed, he does not legislate and suffers the policy cost of the status quo. If the challenger is elected, his expected second-period payoff is

$$D \equiv -\lambda^2(1 - \pi^C \rho)$$

As when he is reelected, if the policy window remains shut he suffers policy loss. If the policy window opens, the status quo is replaced only if his replacement is a zealot. However, he does not have to pay a cost k to replace the status quo himself if the policy window opens.

We now turn to the legislator's strategy in the first period. We begin by constructing separating equilibria in which the slacker does not legislate while the zealot legislates for at least one round then consider pooling equilibria in which neither type legislates.

M.2.1 Separating Equilibrium

In a separating equilibrium, the zealot exerts a positive level of legislative effort while the slacker does not legislate. The voter learns the incumbent's type if she observes the legislative phase and reelects only the zealot. If the voter does not observe the legislative phase, she retains her prior belief about the incumbent's type and reelects the incumbent if and only if $\pi^I \geq \pi^C$. If $\pi^I \geq \pi^C$, we refer to the voter as *trusting*. Conversely, if $\pi^I < \pi^C$ we call the voter *skeptical*.

For a separating equilibrium to exist, the slacker must be unwilling to imitate the zealot to get reelected. This requires that his equilibrium payoff from doing nothing exceeds his payoff from legislating as much as the zealot. With skeptical voters, the slacker always loses the election in a separating equilibrium. Either the voter observes no effort and learns that the legislator is a slacker or she does not observe the legislative phase and falls back on her prior belief that the challenger is better. This yields an equilibrium payoff zero. If instead the slacker devotes $n_1 > 0$ to legislation and is reelected if the voter observes n_1, he earns a payoff of $\zeta b - n_1 k$. With skeptical voters therefore separating equilibrium requires

$$0 \geq \zeta b - n_1 k$$

if the the zealot chooses n_1 in equilibrium.

With trusting voters, the slacker loses the election if the voter observes his lack of effort but wins the election if the voter does not observe the legislative phase. His equilibrium payoff in this case is $(1-\zeta)b$. If instead he devotes $n_1 > 0$ to legislation and is reelected if the voter observes n_1, he is guaranteed to be reelected and earns a payoff of $b - n_1 k$. With trusting voters, separating requires

$$(1-\zeta)b \geq b - n_1 k$$

if the the zealot chooses n_1 in equilibrium.

With both trusting and skeptical voters therefore, separating equilibrium requires that the zealot choose

$$n_1 \geq \frac{\zeta b}{k}$$

Let

$$n^* \equiv \lceil \frac{\zeta b}{k} \rceil$$

where $\lceil \cdot \rceil$ is the ceiling function that returns the smallest integer greater than or equal to its argument. In the first period, legislative effort has no effect on policy. Its only purpose for the zealot is to signal to the voter. The zealot therefore chooses the least amount of legislative effort necessary to distinguish himself from the slacker in a separating equilibrium, n^*.

In order for a separating equilibrium to exist, the zealot must be willing to exert n^* to be reelected. With trusting voters, the zealot is reelected regardless of how much effort he expends if the voter does not observe legislative activity. His equilibrium payoff is therefore

$$W - n^* k$$

If he chooses not to legislate, he loses the election only if the voter observes his lack of legislative effort. His payoff from no legislation is therefore

$$(1-\zeta)W - \zeta D$$

Comparing the two payoffs and plugging $\lceil \frac{\zeta b}{k} \rceil$ in for n^*, it follows that a separating equilibrium exists with trusting voters only if

$$\frac{\zeta b}{k} + \frac{\zeta \rho[\lambda^2(1-\pi^C) - k]}{k} \geq \lceil \frac{\zeta b}{k} \rceil \tag{M1}$$

If the voter is skeptical, then in the separating equilibrium the zealot is reelected only if the voter observes his legislative effort. This yields an equilibrium payoff of

$$\zeta W - (1-\zeta)D - n^* k$$

If instead he does not legislate, he loses the election for sure. His payoff from this action is D. Comparing these two payoffs reveals that (M1) is a necessary condition for a separating equilibrium for skeptical voters as well.

Proposition M1 (Separating Equilibrium) *There exists a separating equilibrium in which incumbent zealots legislate for $n^* > 0$ rounds and slackers legislate for no rounds if and only if* (M1) *is satisfied.*

Proof of Proposition M1: We first provide a complete definition of the equilibrium that generates the outcomes described in the Proposition:

Let h^{C*} denote a legislative action sequence of length n^* and denote elements of this sequence $a_1^{\iota *}, x_1^{\iota *}, v_1^{\iota *}$. The following is a separating equilibrium if and only if (M1) is satisfied

- second-period equilibrium strategies are as specified in Lemma M1
- $w^*(h^V) = \begin{cases} 1 & \text{if} \quad \mu(h^V) \geq \pi^C \\ 0 & \text{otherwise} \end{cases}$
- $p_1^*(h^P; r) = 1 \quad \forall\, x_1^\iota$
- $b_1^*(h^L; z) = \begin{cases} (1, x_1^{\iota *}) & \text{if} \quad \chi(h^L) \leq n^* \\ (0, \emptyset) & \text{otherwise} \end{cases}$

- $b_1^*(h^L; s) = (0, \emptyset) \quad \forall h^L$
- $\mu(h^V) = 1$ if $\chi(h^V) \geq n^*$
- $\mu(h^V) = 0$ if $\chi(h^V) = 0$
- $\mu(\emptyset) = \pi^I$
- $\mu(h^V) < \pi^C$ if $0 < \chi(h^V) < n^*$

We now prove the proposition. The argument above establishes that (M1) is a necessary condition for a separating equilibrium to exist. We now show that (M1) is a sufficient condition for a separating equilibrium. Lemma M1 establishes that second-period strategies are compatible with equilibrium. Given these second-period strategies, the voter's strategy is optimal given her beliefs. To establish that her beliefs are consistent with the intuitive criterion, first note that in a separating equilibrium, if the voter observes h^{C*}, $\mu(h^{C*}) = 1$ by Bayes rule. Similarly if she observes $h^C = \emptyset$, $\mu(\emptyset) = \pi^I$ by Bayes' rule. Bayes' rule also pins down her equilibrium beliefs for $\chi(h^C) = 0$ in which case $\mu(h^C) = 0$. Off path, because n^* is the minimum number of rounds required to separate, both types strictly benefit if reelected by exerting $n < n^*$. Therefore the intuitive does not restrict beliefs after any h^C such that $\chi(h^C) < n^*$. From the definition of n^*, the slacker can never benefit from legislating more than n^* times. Therefore for any h^C such that $\chi(h^C) \geq n^*$, $\mu(h^C) = 1$ survives the intuitive criterion.

Given the voter's strategy and beliefs, there exist no profitable deviations from equilibrium for the slacker. Legislating for $n < n^*$ rounds results in the election of the challenger. Legislating for $n \geq n^*$ ensures reelection but the cost of legislating exceeds the office benefits in the next period. The zealot similarly cannot do better than equilibrium. Legislating less than n^* rounds leads to the election of the challenger which makes him worse off. Legislating for more than n^* rounds results in reelection but at a higher cost than in equilibrium.

Finally, note that vetoing all legislation is a weakly dominated strategy for the incumbent president. Therefore his strategy of vetoing all legislation is an equilibrium strategy. □

If (M1) is satisfied, there exists a large multiplicity of separating equilibria because the specific n^* bills that the zealot passes and the president vetoes are inconsequential to the zealot's payoff. In all separating equilibria, the slacker never legislates and the president never signs a bill. In all equilibria, the zealot legislates. The content of the legislation sent to the president can vary drastically across all equilibria. However, in all separating equilibria, the zealot legislates *exactly* n^* times, just enough to separate from the slacker.

M.2.2 Pooling Equilibrium

We now consider pooling equilibria. In a pooling equilibrium, the voter does not learn anything new about the incumbent legislator after observing legislative action. It follows that for trusting voters, incumbent legislators are reelected in a pooling equilibrium. With skeptical voters, challengers are elected. We focus on a pooling equilibrium in which legislators do not legislate in period one. While other pooling equilibria may exist in which legislators legislate for a positive number of rounds, a pooling equilibrium without legislation maximizes each type's equilibrium payoff. Accordingly, the existence of a pooling equilibrium without legislation is a necessary condition for the existence of any pooling equilibrium.

It is straightforward to check that with trusting voters, a pooling equilibrium always exists. For neither type of legislator does legislating in the first period have a positive effect on their payoff. Slackers only care about being retained in order to earn the office benefit in the first period. While zealots care about about policy in addition to office benefit, the policy gains from retention arise solely through legislative action in the second period. First-period legislation only harms the zealot. Because both types are retained with probability one in equilibrium, neither type has any incentive to legislate.

With skeptical voters, challengers are elected in a pooling equilibrium. In this case the equilibrium payoff for the slacker is zero. If the slacker instead chooses $n_1 > 0$ and the voter reelects the legislator when she observes legislative activity, his payoff is $\zeta b - n_1 k$. The slacker is therefore willing to legislate as many as $\lfloor \frac{\zeta b}{k} \rfloor$ times if doing so ensures his reelection where

$\lfloor \cdot \rfloor$ is the floor function which returns the largest integer less than or equal to its argument.

The zealot earns a pooling equilibrium payoff of D. If he is reelected after the voter observes n_1, the zealot earns an expected payoff of

$$\zeta W - (1-\zeta)D - n_1 k$$

He is therefore willing to legislate as many as

$$\lfloor \frac{\zeta b}{k} + \frac{\zeta\rho[\lambda^2(1-\pi^C) - k]}{k} \rfloor$$

rounds to get reelected.

Pooling equilibrium requires that the slacker must be willing to exert as much legislative effort to ensure retention as the zealot:

$$\lfloor \frac{\zeta b}{k} + \frac{\zeta\rho[\lambda^2(1-\pi^C) - k]}{k} \rfloor \leq \lfloor \frac{\zeta b}{k} \rfloor \tag{M2}$$

This is a consequence of the intuitive criterion. If (M2) fails, the zealot can signal his type to the voter by legislating more than the voter knows the slacker is willing to legislate if doing so were to guarantee reelection. Under the intuitive criterion, the only reasonable belief for the voter is that the legislator is a zealot.

Proposition M2 (Pooling Equilibrium) *There exists a pooling equilibrium in which neither type of incumbent legislator legislates if and only if either*

- *voters are trusting*
- *voters are skeptical and* (M2) *is satisfied*

Proof of Proposition M2: We first provide a complete statement of the equilibrium that generates the equilibrium outcomes described in the Proposition:

The following is a pooling equilibrium if and only if (i) $\pi^I \geq \pi^C$ or (ii) $\pi^I < \pi^C$ and (M2) is satisfied:

- second-period equilibrium strategies are as specified in Lemma M1
- $w^*(h^V) = \begin{cases} 1 & \text{if} \quad \mu(h^V) \geq \pi^C \\ 0 & \text{otherwise} \end{cases}$
- $p_1^*(h^P; r) = 1 \quad \forall\, x_1^\iota$
- $b_1^*(h^L; z) = b_1^*(h^L; s) = (0, \emptyset) \quad \forall h^L$
- $\mu(h^V) = \pi^I \quad \text{if} \quad h^V = \{0, \emptyset\} \text{ or } \quad h^V = \emptyset$
- $\mu(h^V) < \pi^C$otherwise

We now prove the proposition. We first show that the conditions are sufficient for the existence of a pooling equilibrium. Second-period strategies are consistent with equilibrium by Lemma M1. Given these second-period strategies, the voter's strategy is optimal given her beliefs. On the equilibrium path, the voter observes either $h^C = \emptyset$ or $h^C = \{0, \emptyset\}$. By Bayes' rule $\mu(\emptyset) = \mu(\{0, \emptyset\}) = \pi^I$. Off path, if $\pi^I \geq \pi^C$ both types are strictly worse off legislating for more than zero rounds compared to equilibrium. Therefore $\mu(h^C) < \pi^C$ survives the intuitive criterion. If $\pi^I < \pi^C$ and (M2) is satisfied, then for any number of rounds the slacker can strictly benefit from legislating for, the zealot also strictly benefits. Therefore $\mu(h^C) < \pi^C$ survives the intuitive criterion for any h^C such that $\chi(h^C) \geq 1$.

The slacker and zealot are retained at no cost of legislation in the first period if $\pi^I \geq \pi^C$. Therefore no deviation can possibly raise their payoff in this case. If $\pi^I < \pi^C$ and (M2) is satisfied, any deviation results in the election of the challenger given the voter's strategy. This makes both types strictly worse off than in equilibrium where they lose the election without expending costly legislative effort.

As shown earlier, it is a weakly dominant strategy for the incumbent president to veto all legislation in the first period. Therefore conditions (i) or (ii) are sufficient for a pooling equilibrium to exist.

We now show that (i) or (ii) are necessary for a pooling equilibrium. Assume both conditions fail so that $\pi^I < \pi^C$ and (M2) fails. Because (M2) fails, there is some positive number of rounds n that the zealot is willing to legislate for if retained that the slacker is not. Under the intuitive criterion, for any h^C such that $\chi(h^C) = n$, all beliefs other than $\mu(h^C) = 1$ are ruled out. Therefore the voter must retain the legislator at any information set at which she observes such an h^C. This creates a profitable deviation for the zealot from the pooling equilibrium. Therefore (i) or (ii) are necessary for a pooling equilibrium to exist. $\square$

Note that if voters are trusting, both types of legislators get to benefit from voter good faith without having to legislate at all in order to be reelected. If on the other hand voters are skeptical, both types lose their office in a pooling equilibrium. The zealot would like to separate from the slacker by exerting some amount of legislative effort to message to the voter but when (M2) is satisfied, the amount of legislation necessary to effectively message is too much for the zealot to be willing to carry out.

M.3 Equivalence of Results

The conditions that determine whether a separating or pooling equilibrium exist are equivalent to those in the baseline model. Our results for the prevalence and extent of messaging in the micro-founded model are therefore equivalent to those we obtain in the baseline model. Moreover, first- and second-period equilibrium outcomes are equivalent in both types of equilibria to their corresponding equilibria in the baseline model. It follows that our welfare result for the baseline model obtains in the micro-founded model as well.