The Reputation Politics of the Filibuster: Appendix

Daniel Gibbs*

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This Appendix is organized as follows. Section 1 formally defines the equilibrium concept. Section 2 provides Lemmas not stated in the main text that describe general properties of equilibrium. I use the results from Section 2 to prove results from main text in Section 3. Section 4 characterizes the model's equilibrium for the case not studied in the main text, $\gamma_M = H$ and $c_M \in (\frac{\alpha}{2}, \alpha)$.

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*Department of Political Science, Washington University in St. Louis. Email: gibbsd@wustl.edu.

1 Solution Concept

The solution concept is refinement of perfect Bayesian equilibrium. A perfect Bayesian equilibrium is a profile of strategies and beliefs

$$\sigma = \langle \sigma_b, \sigma_f, \sigma_w, \mu_O, \mu_M \rangle$$

such that σ_b, σ_f, and σ_w are sequentially rational and μ_O and μ_M satisfy

1. Common Beliefs
   - Opposition, MC, and OC's posterior belief about the majority is $\mu_M(\cdot)$ at each history.
   - Majority, MC, and OC's posterior belief about the opposition is $\mu_O(\cdot)$ at each history.
2. Action-Determined Beliefs
   - $\mu_O(h_0) = \mu_O(\textit{no bill}) = \mu_O(\textit{bill}) = 1/2$
   - $\mu_O(\textit{fight}) = \mu_O(\textit{table}) = \mu_O(\textit{filibuster})$
   - $\mu_M(h_0) = 1/2$
   - $\mu_M(\textit{bill}) = \mu_M(\textit{allow vote}) = \mu_M(\textit{filibuster})$
3. Bayesian Updating

$$\mu_M(\textit{bill}) = \frac{\sigma_b(H)}{\sigma_b(H) + \sigma_b(L)} \text{ if } \sigma_b(H) + \sigma_b(L) > 0,$$

$$\mu_M(\textit{no bill}) = \frac{1 - \sigma_b(H)}{2 - \sigma_b(H) - \sigma_b(L)} \text{ if } 2 - \sigma_b(H) - \sigma_b(L) > 0,$$

$$\mu_O(\textit{filibuster}) = \frac{\sigma_f(H)}{\sigma_f(H) + \sigma_f(L)} \text{ if } \sigma_f(H) + \sigma_f(L) > 0,$$

$$\mu_O(\textit{allow vote}) = \frac{1 - \sigma_f(H)}{2 - \sigma_f(H)\sigma_f(L)} \text{ if } 2 - \sigma_f(H) - \sigma_f(L) > 0,$$

$$\mu_M(\textit{fight}) = \frac{\mu_M(\textit{bill})\sigma_w(H)}{\mu_M(\textit{bill})\sigma_w(H) + (1 - \mu_M(\textit{bill}))\sigma_w(L)}$$
$$\text{if } \mu_M(\textit{bill})\sigma_w(H) + (1 - \mu_M(\textit{bill}))\sigma_w(L) > 0,$$

$$\mu_M(table) = \frac{\mu_M(bill)(1-\sigma_w(H))}{\mu_M(bill)(1-\sigma_w(H)) + (1-\mu_M(bill))(1-\sigma_w(L))}$$
$$\text{if } \mu_M(bill)(1-\sigma_w(H)) + (1-\mu_M(bill))(1-\sigma_w(L)) > 0$$

I refine the set of PBE I consider by restricting attention to PBE such that (i) $\sigma_b(H) \geq \sigma_b(L)$, (ii) off-path beliefs satisfy certain conditions, and (iii) party strategies satisfy certain efficiency conditions. I first define the off-path belief conditions. I then define efficiency conditions. I conclude this section by formally defining as *equilibrium* as a PBE that satisfies these conditions.

1.1 Beliefs

I first define the conditions that $\mu_O(\textit{filibuster})$ and $\mu_O(\textit{allow vote})$ must satisfy in the PBE I consider. The conditions reference the set of all $\langle \sigma_f, \mu_O(\textit{filibuster}), \mu_O(\textit{allow vote})\rangle$ such that for a given δ, σ_f is sequentially rational and $\mu_O(\textit{filibuster})$ and $\mu_O(\textit{allow vote})$ satisfy the requirements of PBE. I abuse notation slightly by defining δ more generally than in the main text as

$$\delta \equiv \beta\mu_M(bill)\Big[\sigma_w(H)r_M(\textit{fight}) + (1-\sigma_w(H))r_M(table) - r_M(\textit{allow vote})\Big]$$

As in the main text, $-\delta$ is the net expected opponent-reputation payoff that the opposition receives if it filibusters. The generalization allows for both $\gamma_O \in \{H, L\}$ by replacing μ_M with r_M. It is straightforward to check that Remark 1 holds for this general definition and that $\delta \in [-\frac{\beta}{4}, \frac{\beta}{4}]$. Note that δ is a function of $\langle \sigma_b, \sigma_w, \mu_M\rangle$.

I now define the belief condition formally. For a given $\delta \in [-\beta/4, \beta/4]$, let $\Sigma_O(\delta)$ denote the set of $\langle \sigma_f, \mu_O\rangle$ such that (i) σ_f is sequentially rational given δ and μ_O and (ii) μ_O satisfies PBE belief conditions 1-3.

Definition 1 (Equilibrium Beliefs: $\mu_O(\textit{filibuster})$ and $\mu_O(\textit{allow vote})$) *$\Sigma_O^*(\delta) \subseteq \Sigma_O(\delta)$ is the set of $\langle \sigma_f, \mu_O\rangle \in \Sigma_O(\delta)$ such that*

1. *either*

   (i) $\sigma_f(H) + \sigma_f(L) > 0$,

   (ii) $\sigma'_f(H) = \sigma'_f(L) = 0$ *for all* $\langle \sigma'_f, \mu'_O\rangle \in \Sigma_f(\delta)$, *or*

   (iii) $\mu_O(\textit{filibuster}) = \frac{\sigma'_f(H)}{\sigma'_f(H)+\sigma'_f(L)}$ *for some* $\langle \sigma'_f, \mu'_O\rangle \in \Sigma_f(\delta)$, *and*

2. either

(i) $2 - \sigma_f(H) - \sigma_f(L) > 0$,

(ii) $\sigma'_f(H) = \sigma'_f(L) = 1$ *for all* $\langle \sigma'_f, \mu'_O \rangle \in \Sigma_f(\delta)$, *or*

(iii) $\mu_O(\textit{allow vote}) = \frac{1-\sigma'_f(H)}{2-\sigma'_f(H)-\sigma'_f(L)}$ *for some* $\langle \sigma'_f, \mu'_O \rangle \in \Sigma_f(\delta)$.

I now define the conditions that $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$ must satisfy in equilibrium. The conditions reference the set of all $\langle \sigma_w, \mu_M(\textit{fight}), \mu_M(\textit{table}) \rangle$ for a given $\mu_M(\textit{bill})$ such that σ_w is sequentially rational and $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$ satisfy the requirements of PBE.

Formally, let $\Sigma_w(\mu_M(\textit{bill}))$ denote the set of $\langle \sigma_w, \mu_M(\textit{fight}), \mu_M(\textit{table}) \rangle$ such that (i) σ_w is sequentially rational given $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$, and (ii) $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$ satisfy PBE belief conditions 1-3 given $\mu_M(\textit{bill})$ and σ_w.

Definition 2 (Equilibrium Beliefs $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$)
$\Sigma^*_w(\mu_M(\textit{bill})) \subseteq \Sigma_w(\mu_M(\textit{bill}))$ *is the set of* $\langle \sigma_w, \mu_M(\textit{fight}), \mu_M(\textit{table}) \rangle \in \Sigma_w(\mu_M(\textit{bill}))$ *such that*

1. either

(i) $\mu_M(\textit{bill})\sigma_w(H) + (1 - \mu_M(\textit{bill}))\sigma_w(L) > 0$,

(ii) $\mu_M(\textit{bill})\sigma'_w(H) + (1 - \mu_M(\textit{bill}))\sigma'_w(L) = 0$
for all $\langle \sigma'_w, \mu'_M(\textit{fight}), \mu'_M(\textit{table}) \rangle \in \Sigma_w(\mu_M(\textit{fight}))$, *or*

(iii) $\mu_M(\textit{fight}) = \frac{\mu_M(\textit{bill})\sigma'_w(H)}{\mu_M(\textit{bill})\sigma'_w(H)+(1-\mu_M(\textit{bill}))\sigma'_w(L)}$
for some $\langle \sigma'_w, \mu'_M(\textit{fight}), \mu'_M(\textit{table}) \rangle \in \Sigma_w(\mu_M(\textit{fight}))$,

2. either

(i) $\mu_M(\textit{bill})(1 - \sigma_w(H)) + (1 - \mu_M(\textit{bill}))(1 - \sigma_w(L)) > 0$,

(ii) $\mu_M(\textit{bill})(1 - \sigma'_w(H)) + (1 - \mu_M(\textit{bill}))(1 - \sigma'_w(L)) = 0$
for all $\langle \sigma'_w, \mu'_M(\textit{fight}), \mu'_M(\textit{table}) \rangle \in \Sigma_w(\mu_M(\textit{fight}))$, *or*

(iii) $\mu_M(\textit{table}) = \frac{\mu_M(\textit{bill})(1-\sigma'_w(H))}{\mu_M(\textit{bill})(1-\sigma'_w(H))+(1-\mu_M(\textit{bill}))(1-\sigma'_w(L))}$
for some $\langle \sigma'_w, \mu'_M(\textit{fight}), \mu'_M(\textit{table}) \rangle \in \Sigma_w(\mu_M(\textit{fight}))$.

Finally, I define the conditions that $\mu_M(\textit{bill})$ and $\mu_M(\textit{no bill})$ must satisfy in equilibrium. I focus on equilibria in which the opposition and constituencies do not interpret an unexpected bill as a signal of the majority's type. That is, I focus on equilibria in which $\mu_M(\textit{bill}) = 1/2$

if $\sigma_b(H) = \sigma_b(L) = 0$. The condition on $\mu_M(\textit{no bill})$ is analogous to those in Definitions 1 and 2. The condition references the set of all $\langle \sigma_b, \sigma_w, \mu_M \rangle$ given a fixed $\langle \sigma_f, \mu_O \rangle$ such that σ_b and σ_w are sequentially rational and μ_M satisfies PBE conditions 1-3 and Definition 2.

Formally, let $\Sigma_M(\sigma_f, \mu_O)$ denote the set of $\langle \sigma_b, \sigma_w, \mu_M \rangle$ given σ_f and μ_O such that (i) $\sigma_b(H) \geq \sigma_b(L)$, (ii) μ_M and μ_O satisfy PBE conditions 1-3, (iii) $\langle \sigma_w, \mu_M(\textit{fight}), \mu_M(\textit{table}) \rangle \in \Sigma_w^*(\mu_M(\textit{bill}))$, and (iv) σ_b is sequentially rational.

Definition 3 (Equilibroum Beliefs: $\mu_M(\textbf{\textit{bill}})$ and $\mu_M(\textbf{\textit{no bill}})$)
$\Sigma_M^*(\sigma_f, \mu_O) \subseteq \Sigma_M(\sigma_f, \mu_O)$ is the set of $\langle \sigma_b, \sigma_w, \mu_M \rangle \in \Sigma_M(\sigma_f, \mu_O)$ such that

1. *either*
   - *(i)* $\sigma_b(H) + \sigma_b(L) > 0$, *or*
   - *(ii)* $\mu_M(\textit{bill}) = 1/2$
2. *either*
   - *(i)* $2 - \sigma_b(H) - \sigma_b(L) > 0$,
   - *(ii)* $2 - \sigma'_b(H) - \sigma'_b(L) = 0$ *for all* $\langle \sigma'_b, \sigma'_w, \mu'_M \rangle \in \Sigma_b(\sigma_f, \mu_O)$, *or*
   - *(iii)* $\mu_M(\textit{no bill}) = \frac{1-\sigma'_b(H)}{2-\sigma'_b(H)-\sigma'_b(L)}$ *for some* $\langle \sigma'_b, \sigma'_w, \mu'_M \rangle \in \Sigma_b(\sigma_f, \mu_O)$,

1.2 Efficiency

In this subsection I define the efficiency conditions that an equilibrium must satisfy. I start with a condition for the opposition. For every δ, the set $\Sigma_O^*(\delta)$ consists of all $\langle \sigma_f, \mu_O \rangle$ such that σ_f is sequentially rational and μ_O satisfies the belief condition in Definition 1. For any pair of profiles $\langle \sigma_f, \mu_O \rangle, \langle \sigma'_f, \mu'_O \rangle \in \Sigma_O^*(\delta)$ such that $\sigma'_f \neq \sigma_f$, the efficiency condition rules out $\langle \sigma_f, \mu_O \rangle$ if both types of the opposition prefer $\langle \sigma'_f, \mu'_O \rangle$ to $\langle \sigma_f, \mu_O \rangle$ with at least one strict preference. Preferences are determined by the opposition's expected payoff under each strategy-belief profile evaluated at the history *bill* where the opposition chooses to filibuster or allow a vote. Formally, for any $\langle \sigma_f, \mu_O \rangle \in \Sigma_O(\delta)$, the expected equilibrium payoff for type θ_O of the opposition at history *bill* is

$$\begin{aligned} F_{\theta_O}(\sigma_f) \equiv \;& (1 - \sigma_f(\theta_O))\Big[\alpha r_O(\textit{allow vote}) - \beta \mu_M(\textit{bill}) r_M(\textit{bill})\Big] \\ & + \sigma_f(\theta_O)\Big[\alpha r_O(\textit{filibuster}) - c_O \cdot \mathbb{1}(\theta_O = L) \\ & \qquad - \beta \mu_M(\textit{bill})\Big(\sigma_w(H) r_M(\textit{fight}) + (1 - \sigma_w(H)) r_M(\textit{table})\Big)\Big] \end{aligned}$$

Definition 4 (Efficiency: Opposition) $\Sigma_O^{**}(\delta) \subseteq \Sigma_O^*(\delta)$ *is the set of* $\langle \sigma_f, \mu_O \rangle \in \Sigma_O^*(\delta)$ *such that no* $\langle \sigma_f', \mu_O' \rangle \in \Sigma_O^*(\delta)$ *exists such that* $F_{\theta_O}(\sigma_f') \geq F_{\theta_O}(\sigma_f)$ *for both* $\theta_O \in \{H, L\}$ *with at least one strict inequality.*

I now define an efficiency condition that the majority's strategy must satisfy.

The efficiency condition for the majority consists of two parts. The first part is analogous to the version of Pareto efficiency that the opposition's strategy must satisfy. For a fixed $\langle \sigma_f, \mu_O \rangle$, the set $\Sigma_M^*(\sigma_f, \mu_O)$ consists of all $\langle \sigma_b, \sigma_w, \mu_M \rangle$ such that σ_b and σ_w are sequentially rational and μ_M satisfies the belief conditions in Definition 2 and 3. For any pair of profiles $\langle \sigma_b, \sigma_w, \mu_M \rangle, \langle \sigma_b', \sigma_w', \mu_M' \rangle \in \Sigma_O^*(\delta)$ such that $\sigma_f' \neq \sigma_f$ or $\sigma_f' \neq \sigma_f$, the Pareto efficiency condition rules out $\langle \sigma_b, \sigma_w, \mu_M \rangle$ if both types of the opposition prefer $\langle \sigma_b', \sigma_w', \mu_M' \rangle$ with at least one strict preference. Preferences are determined by the majority's expected payoff under each strategy-belief profile evaluated at the initial history where the majority chooses to introduce a bill or not after learning its type.

The second part of the efficiency condition considers any pair of profiles $\langle \sigma_b, \sigma_w, \mu_M \rangle, \langle \sigma_b', \sigma_w', \mu_M' \rangle \in \Sigma_O^*(\delta)$ such that both types of the majority are indifferent between the two. The second part of the efficiency condition selects one profile over the other if both types introduce a bill with a lower probability with at least one type introducing a bill with a strictly lower probability. If $\sigma_b' = \sigma_b$, the condition selects one profile over the other if both types fight with a lower probability with at least one type fighting with a strictly lower probability.

I now define these conditions formally. For any

$$\langle \sigma_b, \sigma_w, \mu_M \rangle \in \Sigma_M(\sigma_f, \mu_O),$$

the expected equilibrium payoff for type θ_M of the majority is

$$\begin{aligned} B_{\theta_M}(\sigma_b, \sigma_w) \equiv (1 - \sigma_b(\theta_M)) \left(\alpha r_M(\textit{no bill}) - \frac{\beta}{4} \right) \\ + \left(\frac{\sigma_b(\theta_M)}{2} \right) \Big[W_{\theta_M}(\sigma_w) \big(\sigma_f(H) + \sigma_f(L) \big) + \alpha r_M(\textit{bill}) \big(2 - \sigma_f(H) - \sigma_f(L) \big) \\ - \beta \big(\sigma_f(H) r_O(\textit{filibuster}) + (1 - \sigma_f(H)) r_O(\textit{allowvote}) \big) \Big] \end{aligned}$$

where

$$W_{\theta_M}(\sigma_w) \equiv \sigma_w(H)[\alpha r_M(\textit{fight}) - c_M \cdot \mathbb{1}(\theta_M = L)] + (1 - \sigma_w(L)) r_M(\textit{table})$$

is the expected equilibrium payoff at history *filibuster* for type θ_M minus its expected opponent reputation payoff.[1]

Definition 5 (Efficiency: Majority) *$\Sigma^{**}_M(\sigma_f, \mu_O) \subseteq \Sigma^*_M(\sigma_f, \mu_O)$ is the set of $\langle \sigma_b, \sigma_w, \mu_M \rangle \in \Sigma^*_M(\sigma_f, \mu_O)$ such that*

1. *no $\langle \sigma'_b, \sigma'_w, \mu'_M \rangle \in \Sigma^*_M(\sigma_f, \mu_O)$ exists such that $B_{\theta_M}(\sigma'_b, \sigma'_w) \geq B_{\theta_M}(\sigma_b, \sigma_w)$ for both $\theta_M \in \{H, L\}$ with at least one strict inequality, and*

2. *no $\langle \sigma'_b, \sigma'_w, \mu'_M \rangle \in \Sigma_M(\sigma_f, \mu_O)$ exists such that $B_{\theta_M}(\sigma'_b, \sigma'_w) = B_{\theta_M}(\sigma_b, \sigma_w)$ for both $\theta_M \in \{H, L\}$ and either*

   (i) $\sigma'_b(\theta_M) \leq \sigma_b(\theta_M)$ and $\sigma'_w(\theta_M) \leq \sigma_w(\theta_M)$ for both $\theta_M \in \{H, L\}$ with at least one strict inequality or

   (ii) $\sigma'_b(\theta_M) \leq \sigma_b(\theta_M)$ for both $\theta_M \in \{H, L\}$ with at least one strict inequality.

1.3 Definition of Equilibrium

I now define an *equilibrium* formally as a PBE that satisfies the belief and efficiency conditions.

Definition 6 (Equilibrium) *An equilibrium is a profile of strategies and beliefs $\langle \sigma_b, \sigma_f, \sigma_w, \mu_M, \mu_O \rangle$ such that $\langle \sigma_b, \sigma_w, \mu_M \rangle \in \Sigma^{**}_M(\sigma_f, \mu_O)$ and $\langle \sigma_f, \mu_O \rangle \in \Sigma^{**}_O(\delta)$ where δ is a function of $\langle \sigma_b, \sigma_w, \mu_M \rangle$,*

$$\delta = \beta \mu_M(\mathit{bill}) \Big[\sigma_w(H) r_M(\mathit{fight}) + (1 - \sigma_w(H)) r_M(\mathit{table}) - r_M(\mathit{allow\ vote}) \Big]$$

2 Additional Lemmas

In this section I state and prove Lemmas A1-A3 which establish general properties of equilibria. I use these Lemmas to establish the model's main results in the following section.

Lemma A1 *For $\mu_M(\mathit{bill}) \in (0, 1)$, the majority's response strategy satisfies the belief condition in Definition 2 if and only if*

$$\sigma_w(H) = \begin{cases} 0 & \text{if } c_M \leq \alpha \mu_M(\mathit{bill}) \text{ or } \gamma_O = L \\ 1 & \text{if } c_M > \alpha \mu_M(\mathit{bill}) \text{ and } \gamma_M = H \end{cases}$$

[1] In any PBE the majority receives the same expected payoff from its opponent's reputation at history *filibuster* for both of its available actions.

$$\sigma_w(L) = \begin{cases} \left(\frac{\mu_M(bill)}{1-\mu_M(bill)}\right)\left[\frac{\alpha}{c_M} - 1\right] & \text{if } c_M \in (\alpha\mu_M(bill), \alpha) \text{ and } \gamma_M = H \\ 0 & \text{otherwise} \end{cases}$$

Lemma A2 *A MS-Filibuster-MS equilibrium is the only equilibrium in which* $\sigma_b(H) = 1$ *and* $\sigma_b(L) = 0$. *A MS-Filibuster-MS equilibrium exists if and only if* $\gamma_M = H$, $c_M \geq \alpha$, *and*

$$c_O \leq \begin{cases} q + \frac{\alpha}{2} & if \quad \gamma_O = H \\ \tilde{c}_O(q, 0) & if \quad \gamma_O = L \end{cases}$$

Lemma A3 *If* $\gamma_M = L$ *or* $c_M \leq \frac{\alpha}{2}$, *in equilibrium*

$$\sigma_b(H) = \sigma_b(L) = \begin{cases} 1 & \text{if} \quad \rho > 0, \\ 0 & \text{otherwise.} \end{cases}$$

If $\gamma_M = H$ *and* $c_M \geq \alpha$, *in equilibrium* $\sigma_b(H) = 1$ *and*

$$\sigma_b(L) = \begin{cases} 0 & \text{if} \quad \sigma_f(H) = \sigma_f(L) = 1, \\ 1 & \text{otherwise.} \end{cases}$$

To prove Lemmas A1-A3, it is useful to define the function

$$\tau_i(\gamma_i) \equiv \begin{cases} 1 & \text{if} \quad \gamma_i = H, \\ -1 & \text{if} \quad \gamma_i = L. \end{cases}$$

I use Remarks 1 and 2 in several proofs. Remarks 1 and 2 are established in the main text. I repeat them here for reference.

Remark 1

$$\delta = \begin{cases} 0 & \text{if } \mu_M(bill) = 1 \text{ or } \sigma_w(H) = \sigma_w(L) = 0, \\ \frac{\beta}{4} & \text{if } \mu_M(bill) = \frac{1}{2}, \sigma_w(H) = 1, \text{ and } \sigma_w(L) = 0. \end{cases}$$

Remark 2

$$\rho = \begin{cases} 0 & \text{if} \quad \sigma_f(H) = \sigma_f(L) = 1, \\ \left(\frac{1-\sigma_f(L)}{2}\right)\left(q - \frac{\beta}{2(1+\sigma_f(L))}\right) & \text{if} \quad \sigma_f(H) > \sigma_f(L) \quad \text{and} \quad \gamma_O = H. \end{cases}$$

Otherwise, $\rho > 0$.

Proof of Lemma A1: It is sequentially rational for the majority to fight if, and only if,

$$\alpha[\mu_M(\textit{fight}) - \mu_M(\textit{table})]\tau_M(\gamma_M) \geq c_M \cdot \mathbb{1}(\theta_M = L) \tag{1}$$

I first establish the result for $\gamma_O = L$. If both types of the majority table in equilibrium, then $\mu_M(\textit{table}) = \mu_M(\textit{bill})$. Both types prefer to table given this belief if, and only if, $\mu_M(\textit{fight}) \geq \mu_M(\textit{table})$. If $\sigma_w(H) = \sigma_w(L) \in (0,1)$, then $\mu_M(\textit{fight}) = \mu_M(\textit{table}) = \mu_M(\textit{bill})$. But given these beliefs, the low majority strictly prefers to fight. Such a strategy is therefore inconsistent with PBE. If $\sigma_w(H) > \sigma_w(L)$, then $\mu_M(\textit{fight}) > \mu_M(\textit{table})$ by Bayes' rule. Given these beliefs, the high majority strictly prefers to table. The strategy is therefore inconsistent with PBE. Similarly, if $\sigma_w(H) < \sigma_w(L)$, then $\mu_M(\textit{fight}) < \mu_M(\textit{table})$ by Bayes' rule which implies that the high majority strictly prefers to fight. The only two remaining strategies are $\sigma_w(H) = \sigma_w(L) \in \{0,1\}$. If $\sigma_w(H) = \sigma_w(L) = 0$, then $\mu_M(\textit{table}) = \mu_M(\textit{bill})$ by Bayes' rule. Both types prefer to table given this belief if, and only if, $\mu_M(\textit{fight}) \geq \mu_M(\textit{bill})$. Because $\sigma_w(H) = \sigma_w(L) = 0$ is consistent with equilibrium independent of c_M, the belief condition in Definition 2 requires the MC to not interpret *table* as a signal of the majority's type if $\sigma_w(H) = \sigma_w(L) = 1$. Thus only the response strategy profile $\sigma_w(H) = \sigma_w(L) = 0$ satisfies the belief condition in Definition 2 is $\gamma_O = L$.

I now establish the result for $\gamma_O = H$. I first consider strategies in which both $\mu_M(\textit{fight})$ and $\mu_M(\textit{table})$ are defined under Bayes' rule. Note that because only the low majority pays c_M to fight and $\mu_M(\textit{fight}) < \mu_M(\textit{table})$ if $\sigma_w(H) < \sigma_w(L)$, no equilibrium exists in which $\sigma_w(H) < \sigma_w(L)$. Relatedly, if $\sigma_w(H) > \sigma_w(L)$ then $\mu_M(\textit{fight}) > \mu_M(\textit{table})$. This implies that the high majority strictly prefers to fight. Thus $\sigma_w(H) = 1$ if $\sigma_w(H) > \sigma_w(L)$. For $\sigma_w(H) = 1$ and $\sigma_w(H) < 1$, by Bayes' rule $\mu_M(\textit{table}) = 0$ and

$$\mu_M(\textit{fight}) = \frac{\mu_M(\textit{bill})}{\mu_M(\textit{bill}) + (1 - \mu_M(\textit{bill}))\sigma_w(L)}$$

Given these beliefs for $\gamma_M = H$, $\sigma_w(H) = 1$ and $\sigma_w(L) = 0$ is sequentially rational if and only if $c_M \geq \alpha$. A mixed strategy requires (1) to hold with equality for $\theta_M = H$. Equality can be satisfied if and only if $c_M \in (\alpha\mu_M(\textit{bill}), \alpha)$ and

$$\sigma_w(L) = \left(\frac{\mu_M(\textit{bill})}{1 - \mu_M(\textit{bill})}\right)[\frac{\alpha}{c_M} - 1]$$

Two pooling strategies are possible. If $\sigma_w(H) = \sigma_w(L) = 0$, then $\mu_M(\textit{table}) = \mu_M(\textit{bill})$ by Bayes' rule. This strategy is therefore sequentially rational if and only if $\mu_M(\textit{fight}) = \mu_M(\textit{bill})$ or $\alpha = 0$. If $\sigma_w(H) = \sigma_w(L) = 1$, then $\mu_M(\textit{fight}) = \mu_M(\textit{bill})$ by Bayes' rule. The strategy can be made sequentially rational if, and only if, $c_M \leq \alpha\mu_M(\textit{bill})$. If the inequality fails, then the strategy is not optimal for the low type even if $\mu_M(\textit{table}) = 0$.

Having characterized all possible equilibrium strategies and beliefs for $\gamma_M = H$, I now apply the belief condition in Definition 2. For $c_M \le \alpha\mu_M(\textit{bill})$, the only two possible strategies are the two pooling strategies. Under the the belief condition in Definition 2 , $\mu_M(\textit{fight}) = \mu_M(\textit{table}) = \mu_M(\textit{bill})$ off the equilibrium path of play. Thus only $\sigma_w(H) = \sigma_w(L) = 0$ satisfies Definition 2 if $c_M \le \alpha\mu_M(\textit{bill})$.

If $c_M \ge \alpha$, only $\sigma_w(H) = \sigma_w(L) = 0$ and $\sigma_w(H) = 1$, $\sigma_w(L) = 0$ are possible equilibrium strategies. By Definition 2 if $\sigma_w(H) = \sigma_w(L) = 0$, the MC's off-path belief must be $\mu_M(\textit{fight}) = 1$. However, $\mu_M(\textit{fight}) = \mu_M(\textit{bill}) < 1$ is necessary for $\sigma_w(H) = \sigma_w(L) = 0$ to be an equilibrium. Thus only the separating strategy satisfies Definition 2 if $c_M \ge \alpha$.

Similarly, if $c_M \in (\alpha\mu_M(\textit{bill}), \alpha)$, the only possible equilibrium strategies are $\sigma_w(H) = \sigma_w(L) = 0$ and $\sigma_w(H) = 1$, $\sigma_w(L) \in (0, 1)$. The off-path belief associated with $\sigma_w(H) = \sigma_w(L) = 0$ therefore must be $\mu_M(\textit{fight}) > \mu_M(\textit{bill})$. The high majority strictly prefers to deviate from $\sigma_w(H) = 0$ given this belief. Thus only the semi-separating strategy satisfies Definition 2 if $c_M \in (\alpha\mu_M(\textit{bill}), \alpha)$. □

Proof of Lemma A2

Remark 1 establishes that in every equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$, $\delta = 0$. The opposition's equilibrium strategy therefore does not depend on the majority's response strategy in any equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$. The opposition's strategy in every equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ is uniquely characterized in Lemmas 3 and 4 for $\delta = 0$.

I first show that $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ in equilibrium only if $\gamma_M = H$. The high majority's net expected payoff from introducing a bill is $\alpha\tau_M(\gamma_M) + \rho$ in any equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$. Thus if $\gamma_M = L$, the high type's bill-introduction strategy is optimal if and only if $\rho \ge \alpha$. If the low majority deviates, its best possible response to a filibuster is *table*. To see this, note that if the high majority fights with positive probability, $\mu_M(\textit{fight}) = 1$ by Bayes' rule. In order for this to be sequentially rational for the high majority, the MC must believe $\mu_M(\textit{table}) = 1$. Similarly, if $\sigma_w(H) = 0$ the MC must believe $\mu_M(\textit{fight}) = 1$ to prevent the high majority from deviating in response to a filibuster. Thus regardless of its response to a filibuster, the majority receives an own-reputation payoff of 0. The low type is therefore strictly better off tabling and saving the cost of the fight. It follows that if $\gamma_M = L$, both types of the majority have the same net expected payoff from introducing a bill, $\rho - \alpha$. The equilibrium therefore exists only if $\rho = \alpha$ and that both types receive an identical equilibrium expected payoff. Note that if this equality is satisfied, a continuum of equilibria exist in which $\sigma_w(L) = 0$ and $\sigma_w(H) \in [0, 1]$. A MS-Table equilibrium dominates all other equilibria in this continuum under the efficiency condition in Definition 5.

Now recall from Lemma A1 that if $\gamma_M = L$, it is sequentially rational for both types to table in any equilibrium in which $\mu_M(\textit{bill}) \in (0,1)$. By Remark 1, $\delta = 0$ in such an equilibrium. The opposition's strategy is therefore optimal in a Bill-Table and NB-Table if and only if it is optimal in a MS-Table equilibrium for $\gamma_M = L$. In a Bill-Table equilibrium, both types receive a net expected payoff from introducing a bill of $\rho + \alpha/2 - \alpha(1 - \mu_M(\textit{no bill}))$. Note that if $\mu_M(\textit{no bill}) = 0$, a Bill-Table equilibrium exists if $\rho \geq \alpha/2$. Thus if $\rho = \alpha > 0$, a Bill-Table equilibrium and a MS-Table equilibrium exist. The majority's expected equilibrium payoff in the Bill-Table equilibrium strictly exceeds its expected equilibrium payoff in the MS-Table equilibrium if $\rho + \alpha/2 > \rho$. Thus the Bill-Table equilibrium dominates the MS-Table equilibrium under the efficiency condition in Definition 5 if $\alpha > 0$. For $\alpha = \rho = 0$, a NB-Table and MS-Table equilibrium exist. Both types receive an expected equilibrium payoff of 0 in each equilibrium. The NB-Table equilibrium therefore dominates the MS-Table equilibrium under the efficiency condition in Definition 5. This establishes that $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ in equilibrium only if $\gamma_M = H$.

I now show that $\sigma_w(H) = 1$ in every equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$. For $\gamma_M = H$, the high majority's net expected payoff from introducing a bill in an equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ is $\rho + \alpha$. Remark 2 shows that $\rho > -\beta/2$. By assumption, $\alpha \geq \beta$. Thus $\rho + \alpha \geq 0$. If $\sigma_w(H) < 1$, then $\mu_M(\textit{table}) = 1$ by Bayes' rule. Given $\mu_M(\textit{table}) = 1$, the low majority's response strategy is consistent with equilibrium if and only if $\sigma_w(L) = 0$. This implies that the low majority's net expected payoff from introducing a bill is equivalent to the high majority's, $\rho + \alpha \geq 0$. Such an equilibrium is therefore possible only if $\rho = \alpha = 0$. It follows from Remark 2 that if $\rho = \alpha = 0$, then $\sigma_f(H) = \sigma_f(L) = 0$. That is, the equilibrium is a MS-Filibuster equilibrium. For $\alpha = 0$, $\delta = 0$ in every equilibrium. Thus a NB-Filibuster equilibrium also exists which dominates the MS-Filibuster equilibrium under the efficiency condition from Definition 5. This establishes that $\sigma_w(H) = 1$ in every equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$.

Because $\gamma_M = H$ and $\sigma_w(H) = 1$, the low majority's net expected payoff from introducing a bill in an equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ is

$$\rho + \alpha\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2}\right) + \left(\frac{\sigma_f(H) + \sigma_f(L)}{2}\right) \times \begin{cases} \alpha - c_M & \text{if } \sigma_w(L) > 0 \\ \alpha\mu_M(\textit{table}) & \text{if } \sigma_w(L) = 0 \end{cases}$$

Remark 2 and $\alpha \geq \beta$ imply that

$$\rho + \alpha\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2}\right) > 0$$

if $2 - \sigma_f(H) - \sigma_f(L) > 0$. Because the low majority's response strategy must be optimal in equilibrium and $\alpha\mu_M(\textit{table}) \geq 0$,

$$0 \leq \left(\frac{\sigma_f(H)+\sigma_f(L)}{2}\right) \times \begin{cases} \alpha - c_M & \text{if } \sigma_w(L) > 0 \\ \alpha\mu_M(\textit{table}) & \text{if } \sigma_w(L) = 0 \end{cases}$$

The low majority's bill-introduction strategy is therefore optimal only if $\sigma_f(H) = \sigma_f(L) = 1$. Thus every equilibrium in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$ is a MS-Filibuster equilibrium.

In a MS-Filibuster equilibrium, the majority's bill-introduction strategy is optimal if and only if

$$0 = \begin{cases} \alpha - c_M & \text{if } \sigma_w(L) > 0 \\ \alpha\mu_M(\textit{table}) & \text{if } \sigma_w(L) = 0 \end{cases}$$

It follows that if $\sigma_w(L) = 0$, the majority's bill-introduction strategy is optimal only if $\mu_M(\textit{table}) = 0$. Given $\mu_M(\textit{table}) = 0$, $\sigma_w(L) = 0$ is sequentially rational if and only if $c_M \geq \alpha$. Thus a MS-Filibuster-MS equilibrium exists if and only if the conditions stated in Lemma A2 are satisfied.

All that remains to consider now are MS-Filibuster equilibrium in which $\sigma_w(H) = 1$ and $\sigma_w(L) > 0$. If the majority plays this response strategy, the majority's bill-introduction strategy is optimal if and only if $c_M \geq \alpha$. Its response strategy is optimal if and only if $c_M \leq \alpha[1 - \mu_M(\textit{table})]$. The two conditions are simultaneously satisfied if and only if $c_M = \alpha$ and $\mu_M(\textit{table}) = 0$. Such an equilibrium therefore exists only if a MS-Filibuster-MS equilibrium exists. Because $\sigma_w(H) = 1$ in every such equilibrium, the MS-Filibuster-MS dominates all others under the efficiency condition in Definition 5. □

Proof of Lemma A3:

I first prove the result for $\gamma_M = L$ or $c_M \leq \frac{\alpha}{2}$. I restrict attention to equilibria in which (i) $\sigma_b(H) \geq \sigma_b(L)$ and (ii) $\mu_M(\textit{bill}) = 1/2$ if $\sigma_b(H) = \sigma_b(L) = 0$. Thus $\mu_M(\textit{bill}) \geq 1/2$ in every equilibrium. Lemmas A1 and A2 therefore imply that $\sigma_w(H) = \sigma_w(L) = 0$ in every equilibrium if $\gamma_M = L$ or $c_M \leq \frac{\alpha}{2}$. Remark 1 establishes that if $\sigma_w(H) = \sigma_w(L) = 0$, $\delta = 0$. Thus if $\gamma_M = L$ or $c_M \leq \frac{\alpha}{2}$, the opposition's strategy is independent of the majority's bill-introduction strategy. Formally, ρ does not depend on σ_b. It follows that in every equilibrium, both types of the majority receive the same net expected payoff from introducing a bill,

$$\rho + \tau_M(\gamma_M)[\mu_M(\textit{bill}) - \mu_M(\textit{no bill})]$$

In a NB-Table equilibrium, $\mu_M(\textit{bill}) = 1/2$ and $\mu_M(\textit{no bill}) = 1/2$. Thus a NB-Table equilibrium exists for $\gamma_M = L$ or $c_M \leq \frac{\alpha}{2}$ if and only if $\rho \leq 0$. In a NB-Table equilibrium, both types receive an own-reputation payoff of $\alpha/2$. In any equilibrium for $\rho \leq 0$ in which $\sigma_b(H) = \sigma_b(L) > 0$, both types receive an own-reputation payoff of $\alpha/2$. Their equilibrium payoff in this equilibrium is therefore weakly lower than their payoff in the NB-Table equilibrium. The NB-Table equilibrium dominates these alternatives under the efficiency condition from in Definition 5. In any equilibrium for $\rho \leq 0$ in which $\sigma_b(H) > \sigma_b(L)$, equilibrium requires

$$\rho + \tau_M(\gamma_M)[\mu_M(\textit{bill}) - \mu_M(\textit{no bill})] = 0$$

Because both beliefs are determined by Bayes' rule if $\sigma_b(H) > \sigma_b(L)$, the equality cannot be satisfied if $\gamma_M = H$. If $\gamma_M = L$, the low majority's equilibrium own-reputation payoff is less than $\alpha/2$. Both types therefore receive a strictly lower equilibrium payoff compared to the NB-Table equilibrium. Thus if $\rho \leq 0$, only a NB-Table equilibrium exits satisfies Definition 5.

If $\rho > 0$, a Bill-Table equilibrium exists in which $\mu_M(\textit{bill}) = \mu_M(\textit{no bill}) = 1/2$. In a Bill-Table equilibrium, the majority's own-reputation payoff if it introduces a bill is $\alpha/2$. Because $\rho > 0$ and both $\mu_M(\textit{bill}) = \mu_M(\textit{no bill}) = 1/2$ in every equilibrium in which $\sigma_b(H) = \sigma_f(L)$, the Bill-Table equilibrium dominates all others in which $\sigma_b(H) = \sigma_f(L)$ under the efficiency condition from Definition 5. An equilibrium in which $\sigma_b(H) > \sigma_b(L)$ requires

$$\rho + \tau_M(\gamma_M)[\mu_M(\textit{bill}) - \mu_M(\textit{no bill})] = 0$$

Because both beliefs are determined by Bayes' rule if $\sigma_b(H) > \sigma_b(L)$, the equality cannot be satisfied if $\gamma_M = H$. If $\gamma_M = L$, the high type receives an own-reputation payoff that is less than $\alpha/2$. Both types therefore receive a strictly lower equilibrium payoff compared to the Bill-Table equilibrium. Thus if $\rho > 0$, only a Bill-Table equilibrium exists and satisfies Definition 5.

I now establish the result for $\gamma_M = H$ and $c_M \geq \alpha$. I first show that the high majority's equilibrium bill-introduction strategy is $\sigma_b(H) = 1$. Lemmas A1 and A2 imply that $\sigma_w(H) = 1$ and $\sigma_w(L) = 0$ in every equilibrium and that $\mu_M(\textit{fight}) = 1$ and $\mu_M(\textit{table}) = 0$ in every equilibrium. The high majority's net expected payoff from introducing a bill for $\sigma_f(H) > 0$ is therefore

$$\rho + \alpha \cdot \left[\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2} \right) \mu_M(\textit{bill}) + \left(\frac{\sigma_f(H) + \sigma_f(L)}{2} \right) - \mu_M(\textit{no bill}) \right]$$

Notice that for $\sigma_b(H) \geq \sigma_b(L)$, the high majority's net payoff is minimized if $\mu_M(\textit{bill}) = \mu_M(\textit{no bill})$. Given these beliefs, the high majority weakly prefers to not introduce a bill if and only if

$$\rho + \alpha \cdot \left(\frac{\sigma_f(H) + \sigma_f(L)}{2} \right) \leq 0$$

The inequality is satisfied only if $\rho < 0$. Remark 2 implies that $\rho < 0$ only if $\gamma_O = H$ and $\sigma_f(L) < \sigma_f(H) = 1$. The inequality can therefore be satisfied only if $\rho \leq -\frac{\alpha}{2}$. However, Remark 2 shows that $\rho > -\beta/2$. By assumption, $\alpha \geq \beta$. Thus $\sigma_b(H) = 1$ in every equilibrium if $\gamma_M = H$ and $c_M \geq \alpha$.

Because $\sigma_b(H) = 1$, $\sigma_w(L) = 0$, and $\mu_M(table) = 0$ in every equilibrium, the low majority's net expected payoff from introducing a bill is

$$\rho + \alpha \cdot \left[\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2} \right) \left(\frac{1}{1 + \sigma_b(L)} \right) \right]$$

Remark 2 establishes that $\rho = 0$ if $\sigma_f(H) = \sigma_f(L) = 1$. Thus if $\sigma_f(H) = \sigma_f(L) = 1$, any bill-introduction strategy such that $\sigma_b(H) = 1$ and $\sigma_b(L) \in [0, 1]$ is consistent with equilibrium.

If $\sigma_f(L) < 0$, $\sigma_b(L) < 1$ is an equilibrium strategy for the low majority only if

$$\rho + \alpha \cdot \left[\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2} \right) \left(\frac{1}{1 + \sigma_b(L)} \right) \right] \leq 0$$

Remark 2 shows that if $\gamma_O = L$, $\rho > 0$ if $\sigma_f(L) < 1$. Thus if $\gamma_O = L$, the low majority's equilibrium bill introduction strategy is $\sigma_b(L) = 1$ if $\sigma_f(L) < 1$. If $\gamma_O = H$ and $\sigma_f(L) < 1$, the inequality can be satisfied only if $\rho < 0$. From Remark 2, this requires $\sigma_f(H) > \sigma_f(L)$. Lemma 3 implies that if $\gamma_O = H$, then $\sigma_f(H) = 1$ in any equilibrium in which $\sigma_f(H) > \sigma_f(L)$. The low majority's strategy must therefore satisfy

$$\sigma_b(L) \leq 1 + \frac{\alpha}{2\rho} \left(\frac{1 - \sigma_f(L)}{2} \right)$$

From Remark 2, $\rho > -\beta/2$. The equality therefore cannot be satisfied for $\alpha \geq \beta$.

Thus if $c_M \geq \alpha$ and $\gamma_M = H$, $\sigma_b(H) = 1$ in every equilibrium and $\sigma_b(H) = 1$ in every equilibrium in which $\sigma_f(L) < 1$. All that remains is to show that $\sigma_f(L) = 0$ if $\sigma_f(L) = \sigma_f(H) = 1$. If $\sigma_f(L) = \sigma_f(H) = 1$, then any $\sigma_b(L) \in [0, 1]$ is optimal for the low majority. Lemmas 3 and 4 establish that $\sigma_f(H) = \sigma_f(L) = 1$ is the opposition's equilibrium strategy given δ if and only if

$$c_O \leq \begin{cases} q - \delta + \frac{\alpha}{2} & \text{if} \quad \gamma_O = H \\ \tilde{c}_O(q, \delta) & \text{if} \quad \gamma_O = L \end{cases}$$

Note that $q - \delta + \frac{\alpha}{2}$ and $\tilde{c}_O(q, \delta)$ are strictly decreasing in δ for all q. For $\mu_M(bill) = \frac{1}{1+\sigma_b(L)}$, $\sigma_w(H) = 1$, $\sigma_w(L) = 0$, and $\gamma_M = H$, δ is increasing in $\sigma_b(L)$ such that $\delta = 0$ if $\sigma_b(L) = 0$ and $\delta = -\beta/4$ if $\sigma_b(L) = 1$. Thus a Filibuster-MS equilibrium in which the low majority mixes on bill introduction exists only if a MS-Filibuster-MS equilibrium exists. It is straightforward to check that both types of the majority are indifferent between all Filibuster-MS equilibrium.

Thus the MS-Filibuster-MS equilibrium dominates every other Filibuster-MS equilibrium under the efficiency condition in Definition 5. $\square$

3 Proofs of Results in Main Text

I provide proofs of results from the main text in this section. I first prove Lemmas 1-4. I then state and prove general versions of Propositions 1-4 which directly imply Propositions 1-8 and Corollary 1.

3.1 Lemmas 1-4

Lemma 1 *The majority plays a pure strategy bill-introduction strategy in equilibrium. In any equilibrium in which the majority's bill-introduction strategy is $\sigma_b(H) = 1$, $\sigma_b(L) = 0$, its response strategy is $\sigma_w(H) = 1$, $\sigma_w(L) = 0$.*

Proof of Lemma 1:

Immediately implied by Lemmas A1-A3. $\square$

Lemma 2 *If $c_M \leq \frac{\alpha}{2}$, the majority's equilibrium response strategy is $\sigma_w(H) = \sigma_w(L) = 0$. If $c_M \geq \alpha$, its equilibrium response strategy is $\sigma_w(H) = 1$ and $\sigma_w(L) = 0$.*

Proof of Lemma 2:

Lemmas A1 and A2 directly imply that if $\gamma_M = H$, the majority's equilibrium response strategy is $\sigma_w(L) = 0$ and

$$\sigma_w(H) = \begin{cases} 0 & \text{if} \quad c_M \leq \frac{\alpha}{2}, \\ 1 & \text{if} \quad c_M \geq \alpha. \ \square \end{cases}$$

Lemma 3 *For $\gamma_O = H$, if $q < \delta$ and $c_O \leq \frac{\alpha}{2}$ the opposition's equilibrium strategy is AV. If $q \geq \delta$ or $c_O > \frac{\alpha}{2}$, its equilibrium strategy is*

$$\begin{cases} \textit{Filibuster} & \textit{if } c_O \leq q - \delta + \frac{\alpha}{2}, \\ \textit{OSS(L)} & \textit{if } c_O \in (q - \delta + \frac{\alpha}{2}, q - \delta + \alpha), \\ \textit{OS} & \textit{if } c_O \geq q - \delta + \alpha. \end{cases}$$

Proof of Lemma 3

The opposition's net expected opponent-reputation payoff, δ, is defined in the main text as a function of $\mu_M(bill)$, σ_w, and γ_M. It is straightforward to check that $\delta \in [-\frac{\beta}{4}, \frac{\beta}{4}]$. Given δ and $\gamma_O = H$, it is sequentially rational for the opposition to filibuster if and only if

$$q - \delta + \alpha[\mu_O(\mathit{filibuster}) - \mu_O(\mathit{allow\ vote})] - c_O \cdot \mathbb{1}(\theta_O = L) \geq 0 \tag{2}$$

Notice that because only the low opposition suffers $c_O > 0$ from filibustering, $\sigma_f(H) \geq \sigma_f(L)$ in every equilibrium and at least one type must play a pure strategy. I first consider strategies in which $\sigma_f(H) + \sigma_f(L) \in (0, 2)$. If the opposition plays such a strategy in equilibrium, then by Bayes' rule $\mu_O(\mathit{fight}) = \frac{\sigma_f(H)}{\sigma_f(H)+\sigma_f(L)}$ and $\mu_O(\mathit{allow\ vote}) = \frac{1-\sigma_f(H)}{2-\sigma_f(H)-\sigma_f(L)}$. Substituting these beliefs into 2 shows that

- $\sigma_f(H) = 1$ and $\sigma_f(L) = 0$ is sequentially rational if and only if $c_O \geq q - \delta + \alpha$,
- $\sigma_f(H) = 1$ and $\sigma_f(L) \in (0, 1)$ is sequentially rational if and only if $c_O \in (q - \delta + \frac{\alpha}{2}, q - \delta + \alpha)$ and $\sigma_f(L) = \frac{\alpha}{c_O-(q-\delta)} - 1$.

It follows that the only possible non-pooling strategies in equilibrium for $\gamma_O H$ are OS and OSS(L). In a Filibuster equilibrium, $\mu_O(\mathit{filibuster}) = 1/2$ by Bayes' rule. Substituting this belief into Inequality 2 shows that the strategy is sequentially rational if and only if

$$\mu_O(\mathit{allow\ vote}) \leq \frac{q - \delta - c_O}{\alpha} + \frac{1}{2}.$$

Notice that the inequality is satisfied only if $c_O \leq q - \delta + \alpha/2$. Thus $\sigma_f(H) = \sigma_f(L) = 1$ in equilibrium only if $q - \delta \leq \alpha/2$. Thus Filibuster, OSS(L), and OS equilibria partition the set of c_O: Filibuster if $c_O \leq q - \delta + \frac{\alpha}{2}$, OSS(L) if $c_O \in (q - \delta + \frac{\alpha}{2}, q - \delta + \alpha)$, and OS if $c_O \geq q - \delta + \alpha$. The only other possible equilibrium is AV. Because $\delta \leq \frac{\beta}{4}$ and $\beta \leq \alpha$, an equilibrium always exists in which $\mu_O(\mathit{filibuster}) = \frac{1}{1+\sigma_f(L)}$. Thus under the belief condition in Definition 1 in an AV equilibrium the OC's off-path belief is

$$\mu_O(\mathit{filibuster}) = \begin{cases} \frac{1}{2} & if \quad c_O \leq q - \delta + \frac{\alpha}{2} \\ \frac{c_O-(q-\delta)}{\alpha} & if \quad c_O \in (q - \delta + \frac{\alpha}{2}, q - \delta + \alpha) \\ 0 & if \quad c_O \geq q - \delta + \alpha \end{cases}$$

Substituting this belief and $\mu_O(\mathit{allow\ vote}) = 1/2$ into (2) shows that an AV equilibrium satisfies Definition 1 if and only if $c_O \leq \frac{\alpha}{2}$ and $q - \delta \leq 0$.

If $q - \delta \leq 0$ and $c_O \leq q - \delta + \frac{\alpha}{2}$, an AV and Filibuster equilibrium exist. In this case Definition 1 requires that the OC's off-path belief in a filibuster equilibrium is $\mu_O(\mathit{allow\ vote}) = 1/2$. Substituting $q - \delta \leq 0$ and $\mu_O(\mathit{allow\ vote}) = \mu_O(\mathit{filibuster}) = 0$ into (2) establishes that a Filibuster equilibrium survives Definition 1 if and only if $q-\delta > 0$. Thus if $q-\delta \leq 0$ and $c_O \leq q-\delta+\frac{\alpha}{2}$, the equilibrium is AV. If $c_O \leq q-\delta+\frac{\alpha}{2}$ and $q-\delta > 0$, the equilibrium is Filibuster.

If $c_O \in (q-\delta+\frac{\alpha}{2}, \frac{\alpha}{2}]$ and $q-\delta \leq 0$, an AV and OSS(L) equilibrium exist. The opposition's difference in expected payoff under each strategy for each type is given by

$$F_H(OSS(L)) - F_H(AV) = q-\delta+\alpha[\frac{c_O-(q-\delta)}{\alpha}] - \frac{\alpha}{2} = c_O - \frac{\alpha}{2}$$

$$F_L(OSS(L)) - F_L(AV) = q-\delta+\alpha[\frac{c_O-(q-\delta)}{\alpha}] - \frac{\alpha}{2} - c_O = -\frac{\alpha}{2}$$

Thus for all $c_O \in (q-\delta+\frac{\alpha}{2}, \frac{\alpha}{2}]$, if $q-\delta \leq 0$ the AV equilibrium dominates the OSS(L) equilibrium under the efficiency condition from Definition 4. □

Lemma 4 *For $\gamma_O = L$, if $c_O \leq \tilde{c}_O(q,\delta)$, the opposition's equilibrium strategy is Filibuster. If $c_O > \tilde{c}_O(q,\delta)$, its equilibrium strategy is*

$$\begin{cases} AV & \text{if} \quad q \leq \frac{\alpha}{2}+\delta, \\ OSS(H) & \text{if} \quad q \in (\frac{\alpha}{2}+\delta, \alpha+\delta), \\ OS & \text{if} \quad q \geq \alpha+\delta. \end{cases}$$

Proof of Lemma 4

Given δ and $\gamma_O = L$, it is sequentially rational for the opposition to filibuster if and only if

$$q-\delta-\alpha[\mu_O(\mathit{filibuster}) - \mu_O(\mathit{allow\ vote})] - c_O \cdot \mathbb{1}(\theta_O = L) \geq 0 \tag{3}$$

Notice that because only the low opposition suffers $c_O > 0$ from filibustering, $\sigma_f(H) \geq \sigma_f(L)$ in every equilibrium and at least one type must play a pure strategy. I first consider strategies in which $\sigma_f(H)+\sigma_f(L) \in (0,2)$. If the opposition plays such a strategy in equilibrium, then by Bayes' rule $\mu_O(\mathit{fight}) = \frac{\sigma_f(H)}{\sigma_f(H)+\sigma_f(L)}$ and $\mu_O(\mathit{allow\ vote}) = \frac{1-\sigma_f(H)}{2-\sigma_f(H)-\sigma_f(L)}$. Substituting these beliefs into 2 shows that

- $\sigma_f(H) = 1$, $\sigma_f(L) = 0$ is sequentially rational if and only if $q-\delta-\alpha \in [0, c_O]$,
- $\sigma_f(H) = 1$, $\sigma_f(L) \in (0,1)$ is sequentially rational if and only if $c_O \in (q-\delta-\alpha, q-\delta-\frac{\alpha}{2})$ and $\sigma_f(L) = \frac{\alpha}{q-\delta-c_O} - 1$,
- $\sigma_f(H) \in (0,1)$, $\sigma_f(L) = 0$ is sequentially rational if and only if $q-\delta \in (\frac{\alpha}{2}, \alpha)$ and $\sigma_f(H) = 2 - \frac{\alpha}{q-\delta}$

Notice that if $q-\delta \leq \frac{\alpha}{2}$, no strategy in which $\sigma_f(H) > \sigma_f(L)$ is consistent with equilibrium. Thus if $q-\delta \leq \frac{\alpha}{2}$, the equilibrium is either AV or Filibuster. Substituting $\mu_O(\textit{filibuster}) = 1$ and $\mu_O(\textit{allow vote}) = 1/2$ into (3) shows that an AV strategy is consistent with equilibrium if and only if $q-\delta \leq \alpha/2$. Thus if $q-\delta \leq \alpha/2$, the Definition 1 requires the OC's off-path belief in a Filibuster equilibrium to be $\mu_O(\textit{allow vote}) = 1/2$. Substituting these beliefs for $q-\delta \leq \alpha/2$ into (3) implies that if $q-\delta \leq \alpha/2$, a Filibuster equilibrium satisfies Definition 1 if and only if $c_O \leq q-\delta$. It follows that if $q-\delta \leq \alpha/2$ and $c_O > q-\delta$, the equilibrium is AV. For $c_O \leq q-\delta$ and $c_O \leq q-\delta$, because a Filibuster equilibrium is possible, Definition 1 requires the OC's off-path belief in an AV equilibrium to be $\mu_O(\textit{filibuster}) = 1/2$. Because $c_O \leq q-\delta$ implies $q-\delta$, an AV equilibrium does not satisfy Definition 1. Thus if $q-\delta \leq \frac{\alpha}{2}$, the equilibrium is Filibuster if $c_O \leq q-\delta$ and AV if $c_O > q-\delta$.

For $q-\delta \in (\frac{\alpha}{2}, \alpha)$, three types of equilibrium are possible:

1. OSS(H) for all all c_O,
2. OSS(L) if and only if $c_O < q-\delta-\frac{\alpha}{2}$
3. Filibuster if and only if $\mu_O(\textit{allow vote}) \geq \frac{1}{2} - \frac{q-\delta-c_O}{\alpha}$

Under the belief condition in Definition 1, if $q-\delta \in (\frac{\alpha}{2}, \alpha)$ and $c_O \geq q-\delta-\frac{\alpha}{2}$ the OC's off-path belief in a Filibuster equilibrium matches its Bayesian belief in an OSS(H) equilibrium, $\mu_O(\textit{allow vote}) = 1-\frac{q-\delta}{\alpha}$. The Filibuster equilibrium therefore satisfies Definition 1 for $q-\delta \in (\frac{\alpha}{2}, \alpha)$ and $c_O \geq q-\delta-\frac{\alpha}{2}$ if and only if $c_O \leq \frac{\alpha}{2}$. Note that $q-\delta-\frac{\alpha}{2} \leq \frac{\alpha}{2}$. Thus for $q-\delta \in (\frac{\alpha}{2}, \alpha)$ and $c_O \in [q-\delta-\frac{\alpha}{2}, \frac{\alpha}{2}]$, an OSS(H) and Filibuster equilibrium exist. The opposition's difference in expected payoff under each strategy for each type is given by

$$F_H(\textit{Filibuster}) - F_H(OSS(H)) = \frac{\alpha}{2}$$

$$F_L(\textit{Filibuster}) - F_L(OSS(H)) = q-\delta+\frac{\alpha}{2} - c_O - \alpha[\frac{(q-\delta)}{\alpha}] = \frac{\alpha}{2} - c_O$$

Thus for all $c_O \leq \frac{\alpha}{2}$, the Filibuster strategy dominates the OSS(H) strategy under the efficiency condition in Definition 4.

For $q-\delta \in (\frac{\alpha}{2}, \alpha)$ and $c_O < q-\delta-\frac{\alpha}{2}$, a OSS(L) equilibrium is also possible. If the OC adopts its Bayesian belief in the OSS(L) equilibrium, $\mu_O(\textit{allow vote}) = 0$ as its off-path belief in the Filibuster equilibrium, neither type prefers to deviate from the Filibuster equilibrium. Thus for $q-\delta \in (\frac{\alpha}{2}, \alpha)$, the Filibuster equilibrium satisfies Definition 1 for all $c_O \leq \frac{\alpha}{2}$. The opposition's difference in expected payoff between the Filibuster strategy and OSS(L) strategy for each type is given by

$$F_H(\textit{Filibuster}) - F_H(OSS(L)) = \frac{\alpha}{2} - \alpha(1 - \frac{q-\delta-c_O}{\alpha})) = q-\delta-\frac{\alpha}{2} + c_O$$

$$F_L(Filibuster) - F_L(OSS(L)) = q - \delta + \frac{\alpha}{2} - c_O - \alpha = q - \delta - \frac{\alpha}{2} - c_O$$

where $\mu_O(\textit{filibuster}) = \frac{q-\delta-c_O}{\alpha}$ in an OSS(L) equilibrium. Thus for all $c_O < q - \delta - \frac{\alpha}{2}$, the Filibuster strategy dominates the OSS(L) strategy under the efficiency condition in Definition 4. Therefore if $q - \delta \in (\frac{\alpha}{2}, \alpha)$, the equilibrium is Filibuster if $c_O \leq \frac{\alpha}{2}$ and OSS(H) if $c_O > \frac{\alpha}{2}$.

Finally, if $q - \delta \geq \alpha$, three types of equilibrium are possible:

1. OS if and only if $c_O \geq q - \delta - \alpha$,
2. OSS(L) if and only if $c_O \in (q - \delta - \alpha, q - \delta - \frac{\alpha}{2})$,
3. Filibuster if and only if $\mu_O(\textit{allow vote}) \geq \frac{1}{2} - \frac{q-\delta-c_O}{\alpha}$

Note that in both a OS and OSS(L) equilibrium, $\mu_O(\textit{allow vote}) = 0$. Thus under Definition 1, $\mu_O(\textit{allow vote}) = 0$ off path in a Filibuster equilibrium if $c_O \geq q - \delta - \alpha$. A Filibuster equilibrium therefore satisfies Definition 1 if and only if $c_O \leq q - \delta - \frac{\alpha}{2}$. For $c_O > q - \delta - \frac{\alpha}{2}$, the equilibrium is OS. For $c_O < q - \delta - \alpha$ the equilibrium is Filibuster. For $c_O \in [q - \delta - \alpha, q - \delta - \frac{\alpha}{2}]$, an OS and Filibuster equilibrium exist. The opposition's difference in expected payoff between the Filibuster strategy and OS strategy for each type is given by

$$F_H(Filibuster) - F_H(OS) = \frac{\alpha}{2}$$

$$F_L(Filibuster) - F_L(OS) = q - \delta + \frac{\alpha}{2} - c_O - \alpha = q - \delta - \frac{\alpha}{2} - c_O$$

Thus for all for $c_O \in [q - \delta - \alpha, q - \delta - \frac{\alpha}{2}]$, the Filibuster equilibrium dominates the OS equilibrium under the efficiency criterion in Definition 4. For $c_O \in (q - \delta - \alpha, q - \delta - \frac{\alpha}{2})$, an OSS(L) and Filibuster equilibrium exist. I establish above that the Filibuster equilibrium dominates the OS equilibrium under the efficiency criterion from Definition 4 if $c_O < q - \delta - \frac{\alpha}{2}$. Thus if $q - \delta \geq \alpha$, the equilibrium is Filibuster if $c_O \leq q - \delta - \frac{\alpha}{2}$ and OS if $c_O > q - \delta - \frac{\alpha}{2}$. □

3.2 Propositions 1-8 and Corollary 1

Propositions A1-A4 directly imply Propositions 1-4. They generalize Propositions 1-4 for $\gamma_M \in \{H, L\}$. Corollary 1 and Propositions 5-8 follow directly from Propositions 1-4.

Proposition A1 *For $\gamma_O = L$ and either $c_M \leq \frac{\alpha}{2}$ or $\gamma_M = L$, if $c_O \leq \tilde{c}_O(q, 0)$ the equilibrium is NB-Filibuster-Table. If $c_O > \tilde{c}_O(q, 0)$, the equilibrium is Bill-AV-Table if $q \leq \frac{\alpha}{2}$, Bill-OSS(H)-Table if $q \in (\frac{\alpha}{2}, \alpha)$, and Bill-OS-Table if $q \geq \alpha$.*

Proof of Proposition A1:

Lemmas A1 and A2 establish that if $c_M \leq \frac{\alpha}{2}$ or $\gamma_M = L$, in every equilibrium $\sigma_w(H) = \sigma_w(L) = 0$. Thus $\delta = 0$ in every equilibrium. Therefore if $\gamma_O = L$ and either $c_M \leq \frac{\alpha}{2}$ or $\gamma_M = L$, the opposition's unique strategy is characterized by Lemma 4 for $\delta = 0$. From Lemma A3, the equilibrium is a Bill-Table equilibrium if $\rho > 0$ and a NB-Table equilibrium if $\rho \leq 0$. Remark 2 implies that $\rho \leq 0$ if and only if $\sigma_f(H) = \sigma_f(L) = 1$. From Lemma 4, the opposition plays this strategy in equilibrium if and only if $c_O \leq \tilde{c}_O(q, 0)$. The equilibrium is therefore NB-Filibuster-Table if $c_O \leq \tilde{c}_O(q, 0)$. If $c_O > \tilde{c}_O(q, 0)$, the equilibrium is Bill-Table. The opposition's strategy in the Bill-Table equilibrium is characterized in Lemma 4. □

Proposition A2 *For $\gamma_O = L$ and either $c_M \leq \frac{\alpha}{2}$ or $\gamma_O = L$, if $c_O \leq q + \frac{\alpha}{2}$ the equilibrium is NB-Filibuster-Table. If $c_O \in (q + \frac{\alpha}{2}, q + \alpha)$, the equilibrium is NB-OSS(L)-Table if $q \leq c_O[\frac{2\alpha}{\beta} + 1]^{-1}$ and Bill-OSS(L)-Table otherwise. If $c_O \geq q + \alpha$, the equilibrium is NB-OS-Table if $q \leq \frac{\beta}{2}$ and Bill-OS-Table otherwise.*

Proof of Lemma A2:

Lemmas A1 and A2 establish that if $c_M \leq \frac{\alpha}{2}$ or $\gamma_M = L$, in every equilibrium $\sigma_w(H) = \sigma_w(L) = 0$. Thus $\delta = 0$ in every equilibrium. Therefore if $\gamma_O = H$ and either $c_M \leq \frac{\alpha}{2}$ or $\gamma_M = L$, the opposition's unique strategy is characterized by Lemma 3 for $\delta = 0$. From Lemma A3, the equilibrium is a Bill-Table equilibrium if $\rho > 0$ and a NB-Table equilibrium if $\rho \leq 0$. By Remark 2, $\rho = 0$ if $\sigma_f(H) = \sigma_f(L) = 1$. Lemma 3 shows that for $\gamma_O = H$ and $\delta = 0$, $\sigma_f(H) = \sigma_f(L) = 1$ if and only if $c_O \leq q + \alpha/2$. Under these conditions, the equilibrium is NB-Filibuster-Table. For $c_O \in (q + \frac{\alpha}{2}, q + \alpha)$, the opposition's strategy is OSS(L). I show in the main text that for $c_O \in (q + \frac{\alpha}{2}, q + \alpha)$, $\rho \leq \delta$ if and only if $q \leq c_O[\frac{2\alpha}{\beta} + 1]^{-1}$. For $c_O \geq q + \alpha$, the opposition's strategy is OS. I show in the main text that for $c_O \geq q + \alpha$, $\rho \leq 0$ if and only if $q \leq \frac{\beta}{2}$. □

Proposition A3 *For $c_M \geq \alpha$, $\gamma_M = H$, and $\gamma_O = L$, a MS-Filibuster-MS equilibrium exists if, and only if, $c_O \leq \tilde{c}_O(q, 0)$. A Bill-MS equilibrium exists if, and only if, $c_O > \tilde{c}_O(q, \frac{\beta}{4})$. The equilibrium is Bill-AV-MS if $q \leq \frac{\alpha}{2} + \frac{\beta}{4}$, Bill-OSS(H)-MS if $q \in (\frac{\alpha}{2} + \frac{\beta}{4}, \alpha + \frac{\beta}{4})$, and Bill-OS-MS if $q \geq \alpha + \frac{\beta}{4}$.*

Proof of Lemma A3:

Lemma A2 characterizes the necessary and sufficient conditions for a MS-Filibuster-MS equilibrium. Lemmas A1-A3 imply that every other equilibrium is a Bill-MS equilibrium if $c_M \geq \alpha$ and $\gamma_M = H$. From Remark 1, $\delta = \beta/4$ in every Bill-MS equilibrium. Lemma 4 characterizes the opposition's unique equilibrium strategy for $\gamma_O = L$ and $\delta = \beta/4$. Note that because $\tilde{c}_O(q, \delta)$ is strictly increasing in δ, a Bill-Filibuster-MS equilibrium exists only if a MS-Filibuster-MS equilibrium exists. A MS-Filibuster-MS equilibrium therefore dominates a Bill-Filibuster-MS equilibrium under the efficiency condition in Definition 5. Thus if $c_O \leq \tilde{c}_O(q, \delta)$, the equilibrium is MS-Filibuster-MS. If $c_O > \tilde{c}_O(q, \beta/4)$, a Bill-MS equilibrium exists in which $\sigma_f(L) < 1$. Lemma 4 characterizes the opposition's specific strategy. □

Proposition A4 *For $c_M \geq \alpha$, $\gamma_M = H$, and $\gamma_O = H$, a MS-Filibuster-MS equilibrium exists if, and only if, $c_O \leq q + \frac{\alpha}{2}$. A Bill-MS equilibrium exists if, and only if, $q < \frac{\beta}{4}$ or $c_O > \frac{\alpha}{2}$. The Bill-MS equilibrium is*

$$\begin{cases} \text{Bill-AV-MS} & \text{if } c_O \leq \frac{\alpha}{2} \text{ and } q \leq \frac{\beta}{4}, \\ \text{Bill-OSS(L)-MS} & c_O \in (\max\{\frac{\alpha}{2}, q - \frac{\beta}{4} + \frac{\alpha}{2}\}, q - \frac{\beta}{4} + \alpha), \\ \text{Bill-OS-MS} & \text{if } c_O \geq q - \frac{\beta}{4} + \alpha. \end{cases}$$

Proof of Lemma A4:

Lemma A2 characterizes the necessary and sufficient conditions for a MS-Filibuster-MS equilibrium. Lemmas A1-A3 imply that every other equilibrium is a Bill-MS equilibrium if $c_M \geq \alpha$ and $\gamma_M = H$. From Remark 1, $\delta = \beta/4$ in every Bill-MS equilibrium. Lemma 3 characterizes the opposition's unique equilibrium strategy for $\gamma_O = H$ and $\delta = \beta/4$. Note that because $q - \delta$ is strictly increasing in δ, a Bill-Filibuster-MS equilibrium exists only if a MS-Filibuster-MS equilibrium exists. A MS-Filibuster-MS equilibrium therefore dominates a Bill-Filibuster-MS equilibrium under the efficiency condition in Definition 5. Thus if $q \geq \beta/4$ and $c_O \leq q - \beta/4 + \alpha/2$ the equilibrium is MS-Filibuster-MS. If $q < \beta/4$ or $c_O > q - \beta/4 + \alpha/2$, a Bill-MS equilibrium exists in which $\sigma_f(L) < 1$. Lemma 3 characterizes the opposition's specific strategy. □

4 Equilibrium for $\gamma_M = H$ and $c_M \in (\frac{\alpha}{2}, \alpha)$

Lemma A2 establishes that if $c_M < \alpha$, no equilibrium exists in which $\sigma_b(H) = 1$ and $\sigma_b(L) = 0$. The proof of Lemma A2 establishes that if $\gamma_M = H$, the high majority strictly prefers to introduce a bill if $\mu_M(bill) = 1$. Thus no equilibrium in which $\sigma_b(H) > 0$ and $\sigma_b(L) = 0$ exists if $\gamma_M = H$ and $c_M \in (\frac{\alpha}{2}, \alpha)$. The only candidates for equilibrium are therefore

1. $\sigma_b(H) = 1$, $\sigma_b(L) > 0$, $\sigma_w(H) = 1$, $\sigma_w(L) = \frac{1}{\sigma_b(L)}\left(\frac{\alpha}{c_M} - 1\right)$
2. $\sigma_b(H) = \sigma_b(L) = 0$, $\sigma_w(H) = 1$, $\sigma_w(L) = \left(\frac{\alpha}{c_M} - 1\right)$

where equilibrium response strategies follow from Lemma A1.

Given these strategies, the high majority prefers to not introduce a bill only if $\gamma_O = H$ and $\sigma_f(H) > \sigma_f(L)$. To see this, first note that the proof of Lemma A3 shows that $\sigma_b(H) = \sigma_b(L) = 1$ in any equilibrium in which $\sigma_f(H) = \sigma_f(L) = 0$. Now note that $\sigma_b(H) \geq \sigma_b(L)$ implies that $\mu_M(\textit{no bill}) \leq 1/2$, $\mu_M(\textit{bill}) \geq 1/2$, and $\mu_M(\textit{fight}) > 1/2$. Thus if $\sigma_f(H) < 1$, the high majority's expected own-reputation payoff from introducing a bill is strictly greater than its expected payoff from not introducing a bill. All that can deter the high majority from introducing a bill is loss it expects to suffer from opposition signaling. This requires $\gamma_O = L$. Thus if $\gamma_O = L$ or $\sigma_f(H) = \sigma_f(L)$, then $\sigma_b(H) = 1$ and $\sigma_b(L) > 0$.

I now use this result to characterize the model's equilibrium for $\gamma_O = L$.

Lemma A4 *If $\gamma_O = L$, $\gamma_M = H$, and $c_M \in (\frac{\alpha}{2}, \alpha)$, a MSS-Filibuster-Fight equilibrium exists if $c_O \leq \tilde{c}(q; 0)$ where $\sigma_b(L) = \frac{\alpha}{c_M} - 1$. If $c_O > \tilde{c}(q; \hat{\delta}(c_M))$, a Bill-MSS equilibrium exists where $\sigma_w(L) = \frac{\alpha}{c_M} - 1$ and*

$$\hat{\delta}(c_M) \equiv \frac{\beta}{2}\left[\frac{c_M}{\alpha} - \frac{1}{2}\right]$$

is the value of δ for $\sigma_b(L) = \frac{\alpha}{c_M} - 1$. The Bill-MSS equilibrium is Bill-AV-MSS if $q \leq \frac{\alpha}{2} + \hat{\delta}(c_M)$, Bill-OSS(L)-MSS if $q \in (\frac{\alpha}{2} + \hat{\delta}(c_M), \alpha + \hat{\delta}(c_M))$, and Bill-OS-MSS if $q \geq \alpha + \hat{\delta}(c_M)$.

Proof of Lemma A4:

I show above that $\sigma_b(H) = 1$ and $\sigma_b(L) > 0$ in every equilibrium if $\gamma_O = L$. Lemma A1 implies that $\sigma_w(L) = \frac{1}{\sigma_b(L)}\left(\frac{\alpha}{c_M} - 1\right)$ in equilibrium. By Bayes' rule $\mu_M(\textit{bill}) = \frac{1}{1+\sigma_b(L)} \geq 1/2$ and $\mu_M(\textit{allow vote}) = 0$ in every equilibrium. The low majority's net expected payoff from introducing a bill in every equilibrium is therefore

$$\rho + \alpha\left(\frac{2 - \sigma_f(H) - \sigma_f(L)}{2}\right)\left(\frac{1}{1 + \sigma_b(L)}\right) - \alpha\mu_M(\textit{no bill})$$

From Remark 2, if $\gamma_O = L$, $\rho = 0$ if $\sigma_f(H) = \sigma_f(L) = 0$ and $\rho > 0$ otherwise. It follows that in any equilibrium in which $\sigma_f(H) < 1$, $\sigma_b(L) = 1$. If $\sigma_f(H) = \sigma_f(L) = 1$, the low majority weakly prefers to introduce a bill if and only if $\mu_M(\textit{no bill})$. Because $\sigma_b(H) = 1$ in every equilibrium, this belief satisfies the belief condition in Definition 3. Given this belief, any $\sigma_b(L) \in [\frac{c_M}{\alpha} - 1, 1]$ is sequentially rational for the low majority.

Having established that $\sigma_b(L) \in (0, 1)$ is consistent with equilibrium only if $\sigma_f(H) = \sigma_f(L) = 1$, I now characterize conditions under which such an equilibrium exists. Given the majority's strategy and $\gamma_M = H$, δ is strictly decreasing in $\sigma_b(L)$ on $[\frac{c_M}{\alpha} - 1, 1]$. At $\sigma_b(L) = \frac{c_M}{\alpha} - 1$, $\delta = 0$. At $\sigma_b(L) = 1$, $\delta = 0$. Note that $\frac{c_M}{\alpha} - 1$ is strictly and continuously decreasing from 1 to 0 in $c_M \in (\frac{\alpha}{2}, \alpha)$ on $c_M \in (\alpha/2, \alpha)$. The set of possible δ induced by

$\sigma_b(L)$ for a given $c_M \in (\frac{\alpha}{2}, \alpha)$ is therefore $[0, \hat{\delta}(c_M)]$ where

$$\hat{\delta}(c_M) \equiv \frac{\beta}{2}\left[\frac{c_M}{\alpha} - \frac{1}{2}\right]$$

is the value of δ for $\sigma_b(L) = \frac{\alpha}{c_M} - 1$. From Lemma 4, $\sigma_f(H) = \sigma_f(L) = 1$ is the opposition's strategy if and only if $c_O \leq \tilde{c}_O(q, \delta)$. Because $\tilde{c}_O(q, \delta)$ is strictly decreasing in δ for all q, a Filibuster equilibrium in which $\sigma_b(L) > \frac{\alpha}{c_M} - 1$ exists only if a Filibuster equilibrium in which $\sigma_b(L) = \frac{\alpha}{c_M} - 1$ exists. Note that if $\sigma_b(L) = \frac{\alpha}{c_M} - 1$, then $\sigma_w(L) = 1$. I refer to such an equilibrium as a MSS-Filibuster-Fight equilibrium. Under the efficiency condition in Definition 5, a MSS-Filibuster-Fight equilibrium exists and dominates every other possible Filibuster equilibrium whenever a Filibuster equilibrium exists.

Because $\sigma_b(L) = 1$ in any equilibrium in which $\sigma_f(L) > 0$ for $\gamma_O = L$, in any equilibrium in which $\sigma_f(L) < 1$, $\sigma_w(L) = \frac{\alpha}{c_M} - 1$ and $\delta = \hat{\delta}(c_M)$. I refer to such an equilibrium as a Bill-MSS equilibrium. From Lemma 4, if $\gamma_O = L$, $\sigma_f(L) < 1$ is the opposition's strategy if and only if $c_O > \tilde{c}(q, \delta)$. Substituting $\hat{\delta}(c_M)$ into $\tilde{c}(q, \delta)$ and applying Lemma 4 yields the characterization of the model's Bill-MSS equilibrium as a function of q in Lemma A4. □

For $\gamma_O = H$, if $\sigma_f(H) > \sigma_f(L)$, a NB equilibrium possible. This requires the high majority to weakly prefer not to introduce a bill given $\mu_M(\textit{no bill}) = \mu_M(\textit{bill}) = 1/2$ and $\mu_M(\textit{fight}) = 1$. From Lemma 3, if $\gamma_O = H$ the high opposition's strategy is either $\sigma_f(H) = 1$ or $\sigma_f(L) = 0$. The high majority's strategy in a NB equilibrium is therefore sequentially rational if and only if

$$\rho - \frac{\alpha}{2}\left(\frac{1 + \sigma_f(L)}{2}\right) + \frac{\alpha}{1 + \sigma_w(L)}\left(\frac{1 + \sigma_f(L)}{2}\right) \leq 0$$

Given $\mu_M(\textit{bill}) = 1/2$ and $\sigma_w(L) = \frac{\alpha}{c_M} - 1$, the condition can be expressed

$$\left(\frac{1 - \sigma_f(L)}{2}\right)\left[q - \frac{\beta}{2(1 + \sigma_f(L))}\right] - \left(\frac{1 + \sigma_f(L)}{2}\right)\left(c_M - \frac{\alpha}{2}\right) \leq 0 \tag{4}$$

where $\rho = \left(\frac{1-\sigma_f(L)}{2}\right)\left[q - \frac{\beta}{2(1+\sigma_f(L))}\right]$ from Remark 2. For $\sigma_f(L) = 0$, the condition is

$$q - \frac{\beta}{2} + \left(c_M - \frac{\alpha}{2}\right) \leq 0$$

Notice that if $c_M = \alpha/2$, the condition is satisfied if and only if $q \leq \frac{\beta}{2}$. Recall that this condition is necessary for a NB-OS-Table equilibrium to exist for $c_M \leq \alpha/2$ and γ_O. For $c_M \in (\alpha/2, \alpha)$, the condition is satisfied if and only if

$$c_M \leq \frac{\alpha + \beta}{2} - q$$

Note that $\frac{\alpha+\beta}{2} - q > \alpha/2$ if and only if $q < \beta/2$. Because $\sigma_w(L) = \frac{\alpha}{c_M} - 1$ if $\sigma_b(H) = \sigma_b(L) = 0$, $\delta = \hat{\delta}(c_M)$. From Lemma 3, $\sigma_f(H) = 1$ and $\sigma_f(L) = 0$ is the opposition's equilibrium strategy for $\gamma_O = H$ given $\delta = \hat{\delta}(c_M)$ if and only if

$$c_O \geq q - \frac{\beta}{2}\left[\frac{c_M}{\alpha} - \frac{1}{2}\right] + \alpha$$

Thus a NB-OS-MSS equilibrium exists if and only if $q < \beta/2$, $c_M \leq \frac{\alpha+\beta}{2} - q$, and $c_O \geq q - \frac{\beta}{2}\left[\frac{c_M}{\alpha} - \frac{1}{2}\right] + \alpha$.[2]

The only alternative NB equilibrium is NB-OSS(L)-MSS. The high majority weakly prefers to not introduce a bill in a NB-OSS(L)-MSS equilibrium if and only if Inequality (4) is satisfied for $\sigma_f(L) = \frac{\alpha}{c_O - (q - \hat{\delta}(c_M))} - 1$. Substituting $\sigma_f(L) = \frac{\alpha}{c_O - (q - \hat{\delta}(c_M))} - 1$ into (4) yields the condition

$$\nu(c_O, c_M) \equiv \left[1 - \frac{\alpha}{2[c_O - (q - \hat{\delta}(c_M))]}\right]\left[q - \frac{\beta[c_O - (q - \hat{\delta}(c_M))]}{2\alpha}\right] - \left(\frac{\alpha}{2[c_O - (q - \hat{\delta}(c_M))]}\right)\left(c_M - \frac{\alpha}{2}\right) \leq 0$$

The left-hand side of the inequality can be represented by the function $\nu(c_O, c_M)$. It is straightforward although somewhat tedious to check that $\nu(c_O, c_M)$ has the following properties:

- $\nu(c_O, c_M) > 0$ if $c_M > \frac{\alpha+\beta}{2} - q$.
- For $c_M \in [\frac{\alpha}{2}, \frac{\alpha+\beta}{2}]$, $\nu(c_O, c_M)$ is continuous and strictly decreasing in c_O on the interval $c_O \in [q - \hat{\delta}(c_M) + \frac{\alpha}{2}, q - \hat{\delta}(c_M) + \alpha]$.
- For all $c_M \in [\frac{\alpha}{2}, \frac{\alpha+\beta}{2}]$, $\nu(q + \frac{\alpha}{2}, c_M) \geq 0$ and $\nu(q - \hat{\delta}(c_M) + \frac{\alpha}{2}, c_M) \leq 0$.
- For all $c_O \in [q - \hat{\delta}(\frac{\alpha+\beta}{2} - q) + \frac{\alpha}{2}, q - \hat{\delta}(\frac{\alpha}{2}) + \alpha]$, $\nu(c_O, c_M)$ is increasing in c_M.

It follows that if $c_M \in (\frac{\alpha}{2}, \frac{\alpha+\beta}{2})$, a unique $c_O^*(c_M) \in (q + \frac{\alpha}{2}, q - \hat{\delta}(\frac{\alpha+\beta}{2}) - q) + \alpha)$ exists such that $\nu(c_O, c_M) > 0$ if $c_O < c_O^*(c_M)$, $\nu(c_O, c_M) < 0$ if $c_O^*(c_M)$, and $\nu(c_O^*(c_M), c_M) = 0$. It also follows that $c_O^*(c_M)$ is increasing in c_M. Note from Lemma 3 that the opposition plays the OSS(L) strategy given δ if and only if $c_O \in (q - \delta + \frac{\alpha}{2}, q - \delta + \alpha)$. Thus a Bill-OSS(L)-MSS equilibrium exists for $c_M \in (\frac{\alpha}{2}, \alpha)$ and $\gamma_M = \gamma_O = H$ if and only if $q \leq \frac{\beta}{2}$, $c_M < \frac{\alpha+\beta}{2} - q$

[2]Notice that for $c_M = \frac{\alpha}{2}$, the condition is $c_O \geq q + \alpha$ which is a necessary condition for an OS equilibrium if $\gamma_O = H$ and $c_M \leq \alpha/2$. For $c_M = \alpha$, the condition is $c_O \geq q - \beta/4 + \alpha$ which is a necessary condition for an OS equilibrium if $\gamma_O = \gamma_M = H$ and $c_M \geq \alpha$.

and $c_O \in (c^*_O(c_M), q - \hat{\delta}(c_M) + \alpha)$. Note that if $c_M \approx \alpha/2$, $\hat{\delta}(c_M) \approx 0$. From Lemma 3 and the high majority's incentive compatibility condition above, for $c_M \approx 0$ a NB-OSS(L)-MSS equilibrium exists if and only if $c_O \in (q + \frac{\alpha}{2}, q + \alpha)$ and $q \leq c_O[\frac{2\alpha}{\beta} + 1]^{-1}$, i.e., the same conditions under which a NB-OSS(L)-Table equilibrium exists for $c_M \leq \alpha/2$ and $\gamma_O = H$.

If a NB equilibrium does not exist, then the high majority strictly prefers to introduce a bill in any Bill-MSS equilibrium. To see this, first note that the best possible reputation payoff the majority can receive from *no bill* is $\alpha/2$ which it obtains in a NB-MSS equilibrium. Now note that because $\mu_M(\textit{bill}) = \mu_M(\textit{no bill}) = 1/2$ and $\sigma_w(H) = \frac{\alpha}{c_M} - 1$ in every Bill-MSS equilibrium, $\delta = \hat{\delta}(c_M)$ in every Bill-MSS equilibrium. The opposition's strategy is therefore identical in a Bill-MSS equilibrium and NB-MSS equilibrium. Because $\alpha \geq \beta$, the low majority weakly prefers to introduce a bill in a Bill-MSS form $\mu_M(\textit{no bill}) = 0$ if the high majority strictly prefers to introduce a bill for $\mu_M(\textit{bill}) = 1/2$. Thus if a NB equilibrium does not exist, a Bill-MSS equilibrium exists in which the opposition's strategy is characterized by Lemma 3 for $\delta = \hat{\delta}(c_M)$. Lemma A5 summarizes.

Lemma A5 *For $\gamma_O = H$, $\gamma_M = H$, and $c_M \in (\frac{\alpha}{2}, \alpha)$, if $q \leq \frac{\beta}{2}$, the equilibrium is NB-OS-MSS if $c_M \geq \frac{\alpha+\beta}{2-q}$ and $c_O \geq q - \hat{\delta}(c_M) + \alpha$ and NB-OSS(L)-MSS if $c_M \in (\frac{\alpha}{2}, \frac{\alpha+\beta}{2} - q)$ and $c_O \in [c^*_O(c_M), q - \hat{\delta}(c_M) + \alpha)$. Otherwise, a Bill-MSS equilibrium exists in which σ_f is given by Lemma 3 for $\delta = \hat{\delta}(c_M)$.*