Individual Accountability, Collective Decision-making Appendix

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1 Proof of Remark 1

Remark 1

$R_D(g)$ is increasing in g and $\lim_{g\to\infty} R_D(g) = 1$.

$R_V(g)$ is increasing in g and $\lim_{g\to\infty} R_V(g) = 1$.

$R_{PD}(g) = R_V(g)$.

$R_D(g) > R_V(g)$

$R_V(g') > R_D(g)$ if g' is sufficiently greater than g.

Proof of Remark 1: For each possible realization of group member abilities, each potential member of a governing coalition in an AVE votes for the correct policy with at least probability $q > 1/2$. Theorem 1 in Berend and Sapir (2005) states that if this condition holds for for a group of size n, then the probability that a randomly selected subgroup selects the correct policy by simple majority vote is increasing in the size of the subgroup. The theorem therefore implies that $R_V(1) \leq R_V(3) \leq ... \leq R_V(n)$. It is a well-established result in the uncertain dichotomous choice literature that if the average probability with which a member of a group selects the correct policy exceeds 1/2, then the probability that the group selects the correct policy by simple majority rule approaches 1 as the size of the group approaches infinity. See Theorem 3 in Boland (1989) for a simple proof.

It is straightforward to show that $R_D(g) > R_V(g)$ for $g \geq 3$. Let h_g denote the number of high-ability types in an accountable governing coalition. In each equilibrium, because g is fixed, $Pr(h_g = i)$ is the same in both equilibria for all $i \in \{0, 1, ...g\}$. For all $h_g \in \{\frac{g+1}{2}, ..., g\}$, both equilibria select the correct policy with probability 1. If $h_g = 0$, both equilibria select the correct policy with probability $B(\frac{g+1}{2}, g, q)$. For all $i \in \{1, 2, ...\frac{g-1}{2}\}$, the correct policy is selected with probability 1 in the g-ADE and probability less than one in the g-AVE.

The monotonicity and asymptotic convergence properties of $R_V(g)$ imply that for each $g \geq 3$, a positive odd integer $g' > g$ exists such that $R_V(g') > R_D(g)$. □

2 ADE Beliefs Under Open Deliberation

In a g-ADE under open deliberation,

- for all $i \in C_D$, $\mu_i(y, v, m) = 1$ if $m_i \in \{\mathbf{m}_i(H, 1), \mathbf{m}_i(H, 0)\}$ and $\mu_i(y, v, m) = 0$ if $m_i \in \{\mathbf{m}_i(L, 1), \mathbf{m}_i(L, 0)\}$ for all y, v, and $m_D \in M_D^*$. For all $m_D \notin M_D^*$, $\mu_i(y, v, m) = 0$ for all y and v.

- for all $i \notin C_D$, $\mu_i(I) = 1/2$ for all I.

- for all $i \in N$, $\eta_i(\theta_i, s_i, m) = \eta_i(\theta_i, s_i)$.

3 Voter Beliefs in AVE Under Closed Voting

The second part of Lemma 2 states that $\lim_{g\to\infty} \hat{a}(0, g) = 1/2$ and $\lim_{g\to\infty} \hat{a}(1, g) = 1/2$. In main text I define

$$\hat{a}(0, g) \equiv [1 + \frac{\hat{\lambda}_g(y = 0|\theta_i = H)}{\hat{\lambda}_g(y = 0|\theta_i = L)}]^{-1}$$

and

$$\hat{a}(1, g) \equiv [1 + \frac{\hat{\lambda}_g(y = 1|\theta_i = H)}{\hat{\lambda}_g(y = 1|\theta_i = L)}]^{-1}$$

In the main text I show that

$$\hat{\lambda}_g(y = 0|\theta_i = H) =$$

$$\pi \sum_{i=\frac{g-1}{2}}^{g-1} Pr(h_{g-1} = i) + \sum_{i=0}^{\frac{g-3}{2}} Pr(h_{g-1} = i)[\pi(1-B(\frac{g+1}{2}, g-i-1, 1-q)) + (1-\pi)B(\frac{g+1}{2}, g-i-1, 1-q)]$$

For $q > 1/2$, $\frac{g+1}{2} > (g - i - 1)(1 - q)$. Therefore

$$\lim_{g\to\infty} B(\frac{g+1}{2}, g - i - 1, 1 - q) = 0$$

Thus

$$\lim_{g\to\infty} \hat{\lambda}_g(y=0|\theta_i=H) = \pi$$

I show in the main text that

$$\hat{\lambda}_g(y=0|\theta_i=L) =$$

$$= \pi \sum_{i=\frac{g+1}{2}}^{g-1} Pr(h_{g-1}=i) + \sum_{i=0}^{\frac{g-1}{2}} Pr(h_{g-1}=i)[\pi(1-B(\frac{g+1}{2}, g-i, 1-q)) + (1-\pi)B(\frac{g+1}{2}, g-i, 1-q)]$$

For $q > 1/2$, $\frac{g+1}{2} > (g-i-1)(1-q)$. Therefore

$$\lim_{g\to\infty} B(\frac{g+1}{2}, g-i, 1-q) = 0$$

Thus

$$\lim_{g\to\infty} \hat{\lambda}_g(y=0|\theta_i=L) = \pi$$

It follows that

$$\lim_{g\to\infty} \frac{\hat{\lambda}_g(y=0|\theta_i=H)}{\hat{\lambda}_g(y=0|\theta_i=L)} = 1$$

Therefore $\lim_{g\to\infty} \hat{a}(0,g) = 1/2$. An analogous proof establishes $\lim_{g\to\infty} \hat{a}(1,g) = 1/2$.

4 Voter Beliefs in AVE Under Open Voting

I analyze voter beliefs for $v_i = 0$ and z in the main text. For $v_i = 1$ and z,

$$\tilde{\lambda}_g(v_i=1, z|\theta_i=H) = Pr(v_i=1, z|\theta_i=H) =$$

$$(1-\pi) \sum_{i=0}^{g-1} \frac{1}{2^{g-1}} \binom{g-1}{i} b(z, g-1-i, 1-q)$$

and

$$\tilde{\lambda}_g(v_i = 1, z|\theta_i = L) = Pr(v_i = 1, z|\theta_i = L) =$$

$$\pi(1-q)\sum_{i=0}^{g-1}\frac{1}{2^{g-1}}\binom{g-1}{i}b(z-i, g-1-i, q) + (1-\pi)q\sum_{i=0}^{g-1}\frac{1}{2^{g-1}}\binom{g-1}{i}b(z, g-1-i, 1-q)$$

Thus

$$\frac{\tilde{\lambda}_g(v_i = 1, z|\theta_i = L)}{\tilde{\lambda}_g(v_i = 1, z|\theta_i = H)} = \frac{Pr(v_i = 1, z|\theta_i = L)}{Pr(v_i = 1, z|\theta_i = H)} = q + \frac{\pi(1-q)}{(1-\pi)}\frac{\sum_{i=0}^{g-1}\binom{g-1}{i}b(z-i, g-1-i, q)}{\sum_{i=0}^{g-1}\binom{g-1}{i}b(z, g-1-i, 1-q)}$$

$$= q + \frac{\pi(1-q)}{(1-\pi)}\frac{\sum_{i=0}^{g-1}\binom{g-1}{i}b(g-1-z, g-1-i, 1-q)}{\sum_{i=0}^{g-1}\binom{g-1}{i}b(z, g-1-i, 1-q)}$$

$$= q + \frac{\pi(1-q)^{g-z}}{(1-\pi)(1-q)^z}\frac{\sum_{j=0}^{\frac{g-3}{2}}\binom{g-1}{i}(q^{g-1-j} + q^j) + \binom{g-1}{\frac{g-1}{2}}q^{\frac{g-1}{2}}}{\sum_{j=0}^{\frac{g-3}{2}}\binom{g-1}{i}(q^{g-1-j} + q^j) + \binom{g-1}{\frac{g-1}{2}}q^{\frac{g-1}{2}}}$$

$$= q + \frac{\pi}{1-\pi}(1-q)^{g-2z}$$

Therefore

$$\tilde{a}(1, z, g) = [1 + q + \frac{\pi}{1-\pi}(1-q)^{g-2z}]^{-1}$$

References

Berend, D. and L. Sapir (2005). Monotonicity in the Condorcet jury theorem. *Social Choice and Welfare 24*, 83–92.

Boland, P. (1989). Majority Systems and the Condorcet Jury Theorem. *Journal of the Royal Statistical Society 38*(3), 181–189.